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# **Economic Approaches to Modeling Fertility Determinants**

## **A Selective Review**

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**For the most complete treatment of fertility issues, the analyst should adopt a dynamic-stochastic view of the fertility decision.**

This paper — a product of the Population Advisory Service, Population and Human Resources Department — is part of a larger effort in the department to explore the determinants of contraceptive use. The study was funded by the Bank's Research Support Budget under research project "Impediments to Contraceptive Use in Different Environments" (RPO 675-72). Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Otilia Nadora, room S6-065, extension 31091 (February 1993, 70 pages).

Arroyo reviews critical models of fertility in which the fertility decision is regarded as the outcome of economic choice behavior. He considers, separately, two classes of models. The first are static lifetime fertility models that explain lifetime fertility aggregates and are exemplified by the work of Easterlin-Crimmins (1983), Rosenweig and Schultz (1985), and Montgomery (1987). The second are dynamic stochastic fertility models that have been used to analyze intertemporal or intergenerational decisions on birth-timing and birth-spacing. These are represented by the work of Wolpin (1984), Newman (1988), and the macroeconomic model of Barro and Becker (1989).

Arroyo discusses issues concerning the theoretical specifications and the econometric implementation of these models. With respect to the choice of modeling paradigm, he notes that

static lifetime choice models, while relatively easy to implement, are restrictive in scope. The lifetime decision framework abstracts from the sequential nature of the fertility decision and cannot therefore adequately address how changes in the *time profile* of costs of contraception, wages, incomes, mother's education, or mortality risks affect fertility variables. Static models also cannot explain stylized empirical regularities with time dimensions, such as convergence of fertility rates across countries, the tendency for women to space births as their number of children increases, or the countercyclicality of U.S. fertility to the business cycle.

Arroyo recommends that for the most complete treatment of fertility issues, the analyst should adopt a dynamic-stochastic view of the fertility decision.

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# **Economic Approaches to Fertility Modelling: A Selective Review**

by Cristino R. Arroyo III

## ***Introduction***

The purpose of this paper is to provide an overview and discussion of critical *economic* approaches to the modelling of fertility levels. By an economic approach to fertility modelling we mean any approach that explains fertility outcomes (specifically, birth variables) as the result of *economic decision-making* (the deliberate weighing of costs and benefits), among other things. These would be distinguished from purely noneconomic approaches to fertility, in which fertility levels are fundamentally explained by noneconomic factors (e.g., biological or cultural factors.) This criterion also excludes approaches which include economic factors as part of an explanation of fertility, *but not in any (explicit or implicit) choice-theoretic fashion*. Hence, a model in which lifetime fertility is a function of only uncontrollable socio-economic variables does not fall within the scope of this paper. This categorization also excludes from our discussion pure time-series models where current fertility is modelled largely as a function of past fertility and uncontrollable economic indicators such as the woman's wage, husband's income, age, urbanization. These approaches provide much useful descriptive information about the fertility process and its response to changes in socioeconomic levels but again omit behavioral considerations underlying the active decision to regulate fertility. The exclusion of these models from our review, however, is without prejudice; it should by no means be construed as a reflection on their research value. Many of the models we do review below are based in part or guided by analysis based on noneconomic or nonbehavioral models. It is largely a consequence of a need to narrow down the scope of this paper to manageable levels that we have chosen to focus only on behavioral economic models of fertility.

Despite this narrow categorization there are still numerous papers which are based on choice-theoretic economic models. Existing papers, however, can be usefully classified into one of two categories: static lifetime fertility models and dynamic models of fertility. Under the heading of static lifetime fertility models, we consider as prototypical for purposes of review three models: the framework of Easterlin-Crimmins (1983), which was the organizing paradigm for the National Academy of Sciences study of fertility determinants in less-developed countries (Bulatao and Lee, eds., 1983); the rival microeconomic model of Rosenzweig and Schultz (1985) and (1987); and the important

reformulation of the Easterlin-Crimmins framework carried out by Montgomery (1987). The growing class of dynamic stochastic fertility models are represented in this review by the recent work of Wolpin (1984), Newman (1988), and the macromodel of Barro and Becker (1989).

Each section in this review consists of (i) a summary of the theoretical model under discussion, (ii) accompanying comments regarding the elements of the theory, (iii) a discussion of the how each model has been implemented empirically and, usually, (iv) an evaluation of the strong and weak points of the approach. The discussions are interspersed with technical representations where it was deemed convenient for reference to the original source, or necessary to bring out a point. Effort was made to keep technicalities to a minimum, however as formalisms are the core of any formal model it is generally indispensable to understand them in order to arrive at an informed assessment of a model's strengths and limitations.

Because some of these economic models of fertility were developed out of dissimilar intellectual traditions and for diverse purposes, it is fair to forewarn the reader that this review is written from the vantage point of one who subscribes to the basic modelling framework of neoclassical economics, which would hold that fertility outcomes may be usefully modelled as the end result of constrained utility-maximizing decisions of households. We acknowledge, however, that regardless of one's basic operating paradigm, the scientific validity of the approach rests fundamentally on two criteria: first, that the assumptions of the theory be internally consistent, and secondly, however unrealistic the assumptions may be, the testable implications of the theory should not be rejected by real-world data drawn from a broad range of contexts and sources. As far as the realism of the assumptions made in the models here, our pragmatic view is that this is useful only to the extent that "realistic" assumptions are more likely to lead to testable implications that are difficult to reject with real-world evidence. It is hoped that this Popperian viewpoint does not render the ideas herein useless to the general consumer of fertility research. Nonetheless, should the difference in philosophies be too wide the reader is fortunate to have several good alternative reviews available (see Schultz, 1986; Easterlin, 1986; Cochrane, 1987) and is also invited to consider the original sources and draw her own conclusions.

We shall evaluate the models according to the chronology of their appearance in the literature, beginning with that of Easterlin and Crimmins, 1985 (henceforth, called EC.)

## Static Models

### 1. Easterlin and Crimmins (EC).

#### 1.1. Concepts.

In their 1985 book, The Fertility Revolution, R. Easterlin and E. Crimmins introduced a "supply-demand" model into the fertility determinants literature which hitherto had ignored or underplayed the influence of economizing behavior in the determination of levels of fertility and birth control. EC developed this model by extending the older *proximate determinants* approach to include economic determinants of deliberate fertility control. The proximate determinants approach to fertility ( Davis and Blake, 1956) was an essentially biological theory that posited two basic structural relationships behind fertility outcomes. First, it was held that there exists a direct causal relationship *from* a set of biological states of an individual *to* her lifetime fertility level. These biological states are referred to as the "proximate determinants" of fertility and represent a convenient summary of the *characteristics of the individual* that are relevant to her expected reproductive capacity. Secondly ( and this is the most important component of the approach from a behavioral science standpoint) the proximate determinants approach says that socioeconomic conditions and behavior can influence fertility levels *through altering the proximate determinants* of individual fertility.

One important contribution of EC was to reemphasize the role of the second set of structural relationships, which had become increasingly neglected (though perhaps not unfairly so) in favor of biological proximate determinants. This was accomplished by extending the proximate determinants framework to include behavioral factors derived from the economics tradition. EC identified three determinants of deliberate fertility control: (1) the "supply" of children, defined as *"the number of surviving children a couple would have in the absence of deliberate fertility regulation"* (this is also referred to as the "potential supply" of children; (2) the "demand" for children, defined as *"the number of surviving children parents would want if fertility regulation were costless;"* and (3) the "costs" of fertility regulation, which are defined as *the psychic costs associated with learning about and using specific fertility regulation techniques*. We examine these concepts in reverse order.

#### 1.1.1. Regulation Costs.

With respect to regulation costs, Easterlin and Crimmins remark that

These costs, in turn, depend upon (a) the *attitudes* in society toward the general notion of fertility control and toward specific techniques, and (b) the *degree of access* to fertility control in terms of both the availability of information and the range of specific techniques, and their prices. ( The Fertility Revolution, p.18. Italics mine)

Later EC take the ideal measure of regulation costs to be "data that reflect a household's subjective attitudes toward the use of fertility control, their information about methods of control, and the economic costs of obtaining additional knowledge about techniques of control and purchasing supplies or services needed for control." ( The Fertility Revolution, p.51.) With these various dimensions entering the definition of regulation costs it is clear that EC do not measure regulation costs in the usual sense of a (possibly imputed) monetary measure of required information-gathering activity, psychic discomfort from regulation, and market costs of regulation. It is a more complex index they have in mind even if it is often more convenient to interpret it as a monetary index.

#### 1.1.2. Demand for Children.

Critical to an understanding of the EC framework is an appreciation of their special concepts of demand and supply for children, and how these are different from the usual economic meaning of the terms demand and supply. These terms demand for fertility and supply of fertility are ubiquitous in the research stemming from the EC framework and in the other economic approaches to fertility. From the perspective of building an economic theory of fertility the concepts of demand and supply are clearly important as these summarize the outcome of choice for changes in the relative benefits and the costs of fertility. Unfortunately the terms demand and supply do not mean the same thing across models, and this nonuniform usage has created an unwarranted amount of confusion in the literature.

In the EC framework the demand for children is a consequence of household maximizing behavior under the hypothetical situation where monetary and nonmonetary costs of regulation are zero (this seems to be what they had in mind in their discussion of demand on p.15 of their book.) There is some kind of partial maximizing behavior on the



part of individuals built into their concept of demand, but it is not fully maximizing behavior because EC "demand" is not defined for regulation costs that are nonzero. Hence, the price of regulation has no theoretical bearing on the "demand" of utility-maximizing individuals for children. Contrast this with the meaning of demand in a standard economic choice model where, for individuals to be fully maximized, they determine their levels of demand for all possible price configurations.

One can criticize the EC model on the grounds that it is very difficult to make sense of the implicit assumption that individuals are taken to maximize over only a very small part of the set of price configurations in the economy (in the parlance of statistics, it would be a set of "measure zero"). Granted that one might muster some survey evidence to show that lifetime family size preferences for children are formed independently of regulation costs<sup>1</sup>, such evidence does not imply that a maximizing individual demand for children does not vary with regulation costs (e.g., direct utility functions do not take prices as arguments, but demand functions do.) Additionally, it was completely unnecessary to work with this demand concept, as it is easy enough to define a tractable fertility demand concept which includes regulation costs, as the Montgomery model below does. If anything, in using the EC demand concept one loses a potential way of checking for the consistency of empirical results against preconceptions the survey data supplies one with. If one were to estimate a standard demand equation which includes regulation costs as a right-hand side variable, it would be possible to test if the lifetime demand for children is highly inelastic with respect to regulation cost. We cannot carry out this kind of a test using the EC demand concept because the dependence on regulation costs is only defined for the zero level—any data used to estimate the EC demand function would have to be selected only for zero regulation cost levels, and could not provide a test for the size of the response in demand to changes in regulation costs. It would appear that the EC demand concept was defined as it was in order to take advantage of existing survey data on lifetime family size preferences, and not really to provide a "faithful" representation of the economic concept of demand in the context of the proximate determinants approach.

Since it is difficult to motivate the type of partial maximization of individuals implicit in the EC framework, does this require one to abandon the assumption of maximizing individuals altogether? Not necessarily; correspondence with an economic construct does not of itself make a concept useful nor does noncorrespondence render it useless. Conceivably, one might still take the EC demand concept at face value for now, proceeding on the interpretation that either individuals are non-maximizers, or that they are maximizers but their underlying preferences are such that their ultimate demand for

children is inelastic with respect to the cost of regulation. The second problem with the EC demand concept is that it is not observable from actual outcomes but must be obtained from direct responses to ideal family size questions. Admittedly, there still is considerable bias in the economics profession against the use of answers to hypothetical questions, though economists, on occasion, have relied on survey information and on answers of individuals to retrospective or hypothetical questions.<sup>2</sup> As far as using such responses to measure the EC "demand" for children, it is admissible for as long as in asking the question about ideal lifetime demand for children the analyst (i) controls for regulation costs being zero, (ii) controls for the levels of other variables like the social environment, husband's income, etc., remaining at some fixed levels (e.g., historical levels), and (iii) there is no problem of truthful preference revelation in the survey design. As critics of the EC model have since pointed out (correctly), responses to the question "If you could choose exactly the number of children to have in your whole life, how many would that be?" will not be reliable measures of the demand for children. From the phrasing of the question it is unclear to the respondent that she is to assume that regulation costs are zero and other variables are taken as fixed (possibly at historical levels) when she answers this question. Hence, (i) and (ii) are definitely violated.<sup>3</sup> In addition, whether this particular survey question elicits truthful responses may also be challenged.

### 1.1.3. Supply of Children.

Considering now the notion of the supply of children it is noteworthy, as the authors themselves recognize (The Fertility Revolution, p.50), that EC "supply" is *unobservable* as well. Moreover, like the EC demand concept, the "supply" of children is not regarded as a function of the regulation costs of fertility. These are the two problems associated with the EC supply concept; the latter is mainly a conceptual issue and poses problems of interpretation similar to those created by the EC demand concept. The nonobservability of supply, however, poses problems that are of a more practical nature. Because it is not directly observable it must be defined implicitly in terms of some function of observables. This is where EC appeal to the proximate determinants hypothesis to suggest that "supply" may be proxied for by a linear function of observable proximate determinants, such as the duration of marriage, birth intervals, breastfeeding, etc. Already this begs the question of how completely an individual's "supply" of children is characterized by the choice of these proximate determinants. This issue aside, though, we will show, momentarily, how the supply measure can be calculated econometrically and, as will then become apparent, the

estimability of the EC fertility equation turns out to depend even more crucially upon the direct observability of "demand." This makes the assumed observability of the "demand" for children even more critical for the implementation of this framework.

To provide some perspective, we summarize at this point the overall theoretical structure of the EC framework in the form of a "path":

As evident in the direction of the arrows (indicating causation), the EC framework is a one-way system without any feedback loops that may denote simultaneity effects or lagged effects. The basic EC model is properly regarded as a static model of *lifetime choice*. This is an important and distinguishing feature of the EC framework, which results from the way the demand, supply, and regulation costs have been defined, and which allows for identification of a statistical form of the model.<sup>4</sup>

## 1.2. Econometric Implementation.

### 1.2.1. Model.

Formally, the EC model consists of a three-equation system (in their book EC implement only the first two equations in the system.) To capture the causal links described by the arrows labelled "A" there is the *proximate-determinants equation*:

$$(1.1) \quad S = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_7 x_7 + \alpha_8 z + \epsilon$$

where  $S$  = children ever born

$z$  = use of contraception, in units of time since first use

$x_1, x_2, \dots, x_7$  = seven proximate determinants, namely, duration of marriage in years, first birth interval in months, second birth interval in months, not secondarily sterile, months of breastfeeding in last closed interval, proportion of pregnancy wastage, and  $x_7$ , the proportion of child mortality.

$\epsilon$  = a random disturbance term

To capture the causal relationships labelled "B" in Fig.1, EC posit the *use-equation*:

$$(1.2) \quad z^* = \beta_0 + \beta_1(S_n - S_d) + \beta_2 RC + \nu$$

where  $z^* = z$  defined in (1.1) if  $z > 0$ ,  $= 0$  otherwise

$S_n = (1 - x_7) \cdot N$ , with  $N = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_7 x_7$  in eq.(1.1)  
= "supply of children"

$S_d$  = "demand" for children

$RC$  = regulation costs

$\nu$  = a stochastic disturbance term

Finally, to model the causal link labelled "C" there is posited what may they refer to as the "modernization equation"

$$(1.3) \quad \begin{bmatrix} \mathbf{x} \\ S_d \\ RC \end{bmatrix} = \gamma_0 + \Gamma_1 \cdot \mathbf{m} + \Gamma_2 \cdot \mathbf{c} + \zeta$$

where  $\mathbf{x} = (x_1, x_2, \dots, x_7)'$ , the vector of proximate determinants above

$\mathbf{m}$  = a vector of variables measuring the degree of modernization

$\mathbf{c}$  = a vector of other variables (particularly cultural variables)

$\zeta$  = a stochastic disturbance vector

$\Gamma_1, \Gamma_2$  = parameter matrices to be estimated

In their subsequent (1985) empirical study EC disregard the last equation (1.3)

because data for  $m$  and  $c$  were not extensive and because available data according to EC allowed for a direct estimation of (1.1) and (1.2) without requiring a recursion on (1.3) as well. In a later study (Easterlin, Crimmins, and Osheba, "The Determinants of Fertility Control," in Halloua, Farid, and Cochrane, eds., Demographic Responses to Modernization.) (1.3) is estimated together with (1.1) and (1.2). The original procedure adopted in The Fertility Revolution can be summarized succinctly as follows:

- I. Because  $z$  appears in (1.1) and  $z$  is also function of  $\nu$  ( the error term in (1.2)), if the disturbances  $\epsilon$  and  $\nu$  are correlated direct OLS (Ordinary Least-Squares estimation) on eq.(1.1) will not give unbiased or consistent estimates of the  $\alpha$ 's (i.e., a consistent estimator for  $\alpha$  is one that is arbitrarily close in probability to the true  $\alpha$  in large samples.) To carry out OLS on (1.1) requires substituting  $z$  with an instrumental variable for  $z$  ( i.e., a variable which is correlated with  $z$  but uncorrelated with  $\nu$ .) Such an instrument is given by the expected value of  $z$ ,  $E(z)$ , calculated from the *reduced form* of (1.2):

$$(1.2') \quad z^* = \delta_0 + \delta_1 x_1 + \dots + \delta_7 x_7 + \delta_8 S_d + \delta_9 RC + \bar{\nu}$$

Note that the appearance of  $S_d$  in this reduced form is a consequence of how  $S_d$  was defined—it is assumed to be exogenous (and not simultaneously determined) in the EC framework. Note also that the notation of EC is unfortunately imprecise because the subvector of parameters  $(\delta_1, \dots, \delta_7)$  can be shown to be equal to  $\beta_1(1-x_7) \cdot (\alpha_1, \dots, \alpha_7)$  and is therefore a function of  $x_7$ , the proportion of child mortality. This sort of imprecision can do potential harm to policy inference that is based on reduced forms only like (1.2'). The first step then, is to estimate (1.2') by the Tobit procedure.<sup>5</sup> This allows us to calculate  $E(z)$  for each individual in the sample according to

$$E(z) = \Phi \cdot (\hat{\delta}_0 + \hat{\delta}_1 x_1 + \dots + \hat{\delta}_7 x_7 + \hat{\delta}_8 S_d + \hat{\delta}_9 RC) + \sigma \phi$$

where  $\sigma$  is the Tobit standard error and  $\Phi$  and  $\phi$  are the standard normal c.d.f. and p.d.f. evaluated at the level  $\sigma^{-1} \cdot (\hat{\delta}_0 + \hat{\delta}_1 x_1 + \dots + \hat{\delta}_7 x_7 + \hat{\delta}_8 S_d + \hat{\delta}_9 RC)$  (hatted parameters are the Tobit estimates.)

- II. For each individual observation, replace  $z$  by  $E(z)$  and run OLS on (1.1). This gives consistent estimates of the  $\alpha$ 's.
- III. Construct values of  $S_n = (1-x_7) \cdot N$  using estimates of the  $\alpha$ 's from step II. Use these constructed values in eq.(1.2) and estimate that equation by OLS.

### 1.2.2. Critique of the Empirical EC Model.

The above estimation strategy is, in fact, the appropriate one for implementing the model as EC had outlined it, with its special interpretation of "demand," "supply," and regulation costs, and with its assumptions about the stochastic independence of the disturbances in (1.1)–(1.3). Therefore, as Easterlin (1986) sought to make clear in his rejoinder to Schultz's (1986) critique, the question of appropriate estimation methodology is not the real issue.

Schultz (1986) criticized the estimation strategy for the proximate determinants equation on the grounds that simple OLS on eq.(1.1) gives inconsistent estimates of the true slope coefficients. This, he argues, is because the explanatory variables of (1.1), especially any variable that represents or proxies for contraceptive usage, will generally be correlated with the equation error  $\epsilon$ . This correlation is attributed directly to economic choices in the presence of unobservable variations in individual fecundity or "natural" fertility levels.

Imagine that individuals have different levels of fecundity. Schultz, whose alternative model we will shortly examine, argues that because differences in fecundity are not directly observable to the researcher they will be lumped into the error term  $\epsilon$  of eq.(1.1). Now even if individuals do not know initially what their true fecundity is, if they observe that over time they have higher than average births, they will tend over time to self-select into more rigorous contraceptive regimes. This will mean that such a fecundity component of  $\epsilon$  will be statistically correlated with the equation error of (1.2) above, and will thus be correlated with contraceptive use  $z$ . Since contraceptive use  $z$  is also an explanatory variable in (1.1) for actual births, we have a case where a regressor,  $z$ , is correlated with the equation error, and in such situations straight OLS gives inconsistent estimates of the parameters in (1.1). This led Schultz to different empirical model (see Section 2 below.)

In defence of EC, their original methodology was not OLS but an instrumental-variables procedure *chosen precisely to treat this potential problem of correlation between contraceptive use and the fertility equation error*, which they were quite aware of:

In our exploratory WFS [World Fertility Survey] study, which emphasized simple techniques, we estimated (1a) [ the fertility equation (1.1) above] by ordinary least squares. An objection to this is that the simultaneous nature of the model suggests that the use variable and the disturbance term in (1a) are likely to be correlated. As we shall see in the next chapter, there is evidence to this effect. Hence the present two-stage procedure was adopted in which an instrumental use variable was constructed . . . and employed instead of the observed values of use to estimate (1a). ( The Fertility Revolution, p.41.)

So that part of the Schultz critique which focuses on untreated correlation of contraceptive use with the proximate determinants equation error is amiss. However, Schultz did point out the possibility that the other explanatory variables in  $x$  besides contraceptive use are correlated with the equation error. Schultz also remarks that the static nature of the EC model leaves much to be desired in the treatment of dynamic influences on contraceptive use, which in the EC model are captured very indirectly via adjustments of stock variables ( like expected lifetime excess supply of children.) The possibilities of "non-orthogonality" of the explanatory variables with the equation error should certainly be taken into account in estimation if the researcher has evidence or strong priors that these correlations are likely to be significant. However this criticism can at some level be regarded as a disagreement over the fundamental model that specifies what variables are endogenous (determined by other variables in the model) and what can be treated as exogenous (determined from outside the model), rather than any fundamental disagreement over the strategy of estimation. Easterlin was thus probably correct to regard the point of departure for Schultz's critique as stemming from disagreement about what the "correct" theoretical model to implement is (or should be.) If one were to look closely at the EC model with its idiosyncracies, the estimation procedure adopted for that model is appropriate. Whether the EC model *seriously* misspecifies certain explanatory variables as being exogenous, and therefore statistically uncorrelated with the corresponding equation error is, at bottom, an empirically testable question that requires one to look at the data and think about what one considers a "serious misspecification."<sup>6</sup> Neither side in this debate, however, report *formal tests* of the EC model that would confirm or deny the existence of such remaining correlation, or provide measures for the probable size of parameter inconsistencies due to this omission.

There are still other sources of potential misspecification problems arising from the assumption that the EC demand for children is exogenous or even just statistically

uncorrelated with the error of the reduced-form equation (1.2'). If the lifetime demand for children can, in fact, be measured by asking survey respondents a direct question, then perhaps it is legitimate to treat their answers as exogenous data for  $S_d$  in (1.2'). If, however, we believe that all the only data that one can observe with any reliability is the equilibrium outcome of the interaction between supply forces and demand for children, then it would not be legitimate to use  $S_d$  as an explanatory variable in the reduced form, as the  $S^d$  is determined jointly with supply  $S_n$  (and happens to equal  $S_n$  plus an error that reflects imperfect fertility control—refer back to the definitions of supply and demand given at the start of Section 1.) It becomes necessary, then, to solve a simultaneous model. This fact, together with some other weaknesses mentioned above, led Montgomery (1987) to reformulate the EC framework around this potential problem. But even if we assume in our model that demand is, theoretically, an exogenous variable, it is possible in fact for the actual demand data have considerable statistical correlation with the error term  $\tilde{\nu}$  of (1.2'), possibly because there are omitted explanatory variables that jointly influence the demand for children and the reduced-form error  $\tilde{\nu}$ . For example, "modernization" may lead to greater economic benefits to women's labor force participation and will thus constitute a negative shock to the lifetime demand for children. But it may also lead to shocks to the level of contraceptive usage in a way not captured by any of the explanatory variables  $x$ ,  $S_d$ , or  $RC$ , by say, changing cultural patterns to make contraceptive use at earlier ages more (or less) socially acceptable, or by increasing (or decreasing) average coital frequency, or by increasing (or decreasing) the overall length of stable unions in the society—anything that might possibly shift the intercept parameter  $\delta_0$  of the reduced form (1.2') in a systematic way. If this is the case, then it is necessary to work with the larger model that includes something like equation (1.3), requiring more in the way of analysis and data.

### 1.3. An Evaluation.

Despite the amount of criticism that has been directed against the EC framework it has been pointed out (Montgomery, 1987) that it is a useful and versatile model for organizing one's thinking about the factors affecting fertility and contraceptive use. Its linear design is simple and readily extended and the unambiguous direction of the implied causations simplify somewhat the estimation procedure (which, unfortunately, is needlessly complicated by the EC concepts of demand and supply.) Additionally, once one understands the EC meaning of demand, supply, and regulation costs interpretation of the



results are straightforward. Historically the EC framework's significance lies in reorienting the analytical focus of demographic research in fertility back towards behavioral issues, and with emphasis on the effect of economic factors not just on the demand side but on the supply side as well. Perhaps its biggest current advantage lies in how easily it fits as an analytical tool for the study of KAP-type (knowledge-attitudes-practice) data sets, and there are many of these available. Its biggest practical drawbacks are its static formulation and its ad hoc behavioral specification which leaves it open to many potential misspecification problems. Some of these misspecification problems can contribute to difficulties in the use of the model for policy simulation, but we reserve this for a separate paper on the policy implications of methodological assumptions. The weakest points, conceptually, are the EC concepts of "demand" and "supply," which are awkward and somewhat misleading. In what follows we dispense with their interpretation of these concepts altogether in favor of the standard economic interpretation of demand and supply (which are defined over the full range of costs and prices.) In doing so, we recognize that by itself this does not invalidate the basic approach of EC, but inasmuch as the way EC define demand and supply is not as informative as an economist's standard definition of demand and supply, we hope to lay this terminology to rest.

## **2. Rosenzweig and Schultz (RS).**

### **2.1. Introduction and Concepts.**

An important challenge to the EC paradigm above was raised in the model of M. Rosenzweig and T.P. Schultz ("The Demand for and Supply of Births: Fertility and its Life Cycle Consequences," American Economic Review, 1985), with extensions in other work by the two authors ("Fertility and Investments in Human Capital," Journal of Econometrics, 1985 and "Schooling, Information, and Nonmarket Productivity: Contraceptive Use and its Effectiveness," International Economic Review, 1989.) The model, in its theoretical structure, is an application of the maximization paradigm in economics to fertility and female labor force participation decisions, extending the tradition of the models of household behavior of Becker (1973, 1981). The theoretical model is an optimization problem carried out by individuals subject to, but not fully restricted, by constraints, economic and biological. Hence its main behavioral presumption is that individuals have control over some (not necessarily all) aspects of their own fertility, and make decisions about things like contraceptive use and family size to maximize something, in this case, their welfare as measured by a lifetime utility function

or utility index.

There are those who are uncomfortable with the paradigm of a mathematical representation of preferences. The basic idea is, however, that if preferences over pairwise comparisons of consumption bundles obey some intuitively plausible properties (completeness, continuity, monotonicity, convexity) such a mathematical representation of preferences is guaranteed to exist. The characteristics of the utility function (its form and its parameters) offer a convenient way of describing and summarizing what individual preferences are without really saying how they came to be that way. In this sense the analyst is able to mentally separate the way "conscious" choice behavior operates from the way behavior itself might be "unconsciously" or subconsciously shaped—the "conscious" part of choice behavior is what occurs when individuals make decisions given a utility function, "subconscious" behavior is part of what makes up the black box called the utility function. The standard response to the criticism that economic analysis regards all behavior as a matter of conscious choice is that the construct of the utility function can be extended to include forces, conscious or unconscious, that are imagined to impinge upon the parameters of individual preferences.<sup>7,8</sup> If the problem is that individuals may not be exclusively individualistic, the utility concept can be extended, if necessary, to include certain forms of altruistic behavior (e.g., bequest motives) without really changing the fundamental structure of the choice situation. This idea is pursued, for example, in the model of Barro and Becker (1988, 1989) which is reviewed later.

## 2.2. Rosenzweig and Schultz (1985).

### 2.2.1. Theoretical Specification.

The Rosenzweig-Schultz (1985) theoretical model (RS, hereafter) is a dynamic programming model of individual choice in an environment where there are certain kinds of randomness. Hence it shares many features in common with an earlier theoretical model of Wolpin (1984), which also models fertility choice in a dynamic situation. (In the empirical implementation of their model RS are able to simplify much of the econometrics at the cost of sacrificing all the interesting dynamic properties of the original theoretical model. By contrast, Wolpin's study allows one to address essentially dynamic questions such as birth spacing, birth timing, and intertemporal tradeoffs at the cost of a significantly more complex estimation procedure. We discuss Wolpin's and related work in the section on dynamic stochastic models of fertility.) It is helpful, initially, to think of the

individual in the RS model as a "representative agent." This representative agent maximizes the expected present value of her lifetime utility flows. The discount factor in the present value calculations is the individual's subjective time preference factor  $\beta$  for discounting the value of utility a period away. The expectation of all flows is taken as of a reference time period 0. In each period of time  $t$ , the individual's well-being  $U$  is a smooth, quasiconcave function of the number of current births,  $S_t$ , the current stock of children or family size,  $M_t$ , current consumption of market goods  $C_t$ , current leisure  $L_t$ , and a once-and-for-all preference shock  $\Theta$  realized at the initial period 0. More concisely, the representative individual solves the problem

$$(2.1) \quad \text{maximize } E_0 [\sum_{t=0}^{\infty} \beta^t U(S_t, M_t, C_t, L_t; \Theta)]$$

where  $M_t = M_{t-1} + S_t$ . The expectation  $E_t$  is the individual's subjective expectation at the beginning of period  $t$  of variables dated  $t$  and later, given some information that the individual is presumed to know at time  $t$ . RS do not say explicitly what variables are in this time  $t$  information set but it may be supposed that all variables that are dated  $t$  and earlier are legitimate information variables at time  $t$ . They do make the assumption that the subjective expectations of individuals are equal to expectations calculated based on the analyst's assumed distributions for the random variables in the model (i.e., expectations are "rational.")

As in most economic choice models the individual's freedom to choose is restricted, in this case by two sets of constraints—budget constraints at  $t$ :

$$(2.2) \quad w_t(L^* - L_t) + I_t \geq p_t C_t + q_t M_t + r_t Z_t$$

where  $L^*$  is total time of the individual;  $w_t$  is the wage rate at  $t$ ;  $I_t$  is husband's income at  $t$ ,  $Z_t$  a variable denoting whether contraception was in use; and  $p_t, q_t, r_t$  are the market prices of consumption goods, children, and contraception, respectively. Note that in RS the cost of regulation is purely a market cost, although it is easy enough to extend their framework to include any psychic costs by including  $Z_t$  in the utility function. A dichotomy is thus drawn here between these two kinds of costs in terms of how they enter the model. Compare this with the EC framework in which both attributes of cost are lumped together in total regulation cost  $RC$ , and affect all variables only through their use—equation (1.2).

The second and more relevant constraint for this approach is what RS call the "reproduction function." This is the analog of several proximate determinants equations,

one for each period  $t$ , with somewhat different proximate determinants from the ones RS and earlier researchers had used. Let us now drop the assumption of a representative individual and instead index individuals by  $i=1,\dots,I$ . All variables above would then take on a superscript  $i$  in addition to the time-subscript  $t$ . The  $i$ th individual's reproduction function is defined by RS as

$$(2.3) \quad S_t^i = \mu_0^i + \mu_1(a^i + t) + \mu_2 Z_t^i + \epsilon_t^i$$

where  $S_t^i$  and  $Z_t^i$  are as defined previously,  $a^i + t$  is the  $i$ th individual's age at time  $t$ ,  $\epsilon_t^i$  is a stochastic disturbance,  $\mu_1$  and  $\mu_2$  are fixed coefficients to be estimated, and  $\mu_0^i$  is an individual-varying but time-invariant intercept (also to be estimated). This last term,  $\mu_0^i$ , has the interpretation of being the  $i$ th individual's specific fertility component or individual fecundity, partly known by the individual, but unobservable to the researcher. The other important property of  $\mu_0^i$  is that while it is *correlated with the realized number of births*  $S_t^i$ , it is assumed to be *uncorrelated with preferences* over number of births, family size, consumption, or leisure.

The individual then chooses levels of leisure, market goods, consumption, and contraceptive use in each period to solve (2.1) subject to (2.2) and (2.3). She takes all prices, husband's income, total available time, and the reproduction parameters  $\mu_1$ ,  $\mu_2$ , and possibly  $\mu_0^i$  as given. In addition, as we indicated above, it is assumed that at each time period  $t$  all decisions and outcomes that occurred prior to  $t$  are known, and this includes the time-invariant preference shock  $\Theta$ . Neither  $S_t^i$  nor  $M_t^i$  are known until after period  $t$  when the random shock  $\epsilon_t^i$  is realized.

In principle, this model can be solved for the optimal levels of leisure, consumption, and contraceptive use at each period  $t$ , however, as intimated in the seminal work of Wolpin (1984), closed-form solutions are very difficult to obtain for relatively simple specifications of the utility function  $U$ . Hence, RS concentrate on estimating the reproduction function (2.3) with specific attention to estimating the individual fecundity component, given that individuals are choosing  $Z_t^i$  optimally. They do provide, however, some theoretical results that come out of a two-period version of the above model. The gist of their comparative-static theoretical results are

- i. An increase in  $M_t$ , the current stock of children, that is known to come from

an increase in  $\epsilon_t^i$ , will increase the likelihood of contraceptive use in the succeeding period as long as contraceptive costs are "low enough."

- ii. If individuals know their own fecundity level,  $\mu_0^i$ , an observed increase in family size  $M_t^i$  that is not known to be due entirely to the random shock  $\epsilon_t^i$ , will have the effect of increasing the likelihood of contraceptive use in the succeeding period by more than in (1) above. This is because individuals know that part of the increase in family size is due to their being relatively more fecund.
- iii. If individuals don't know  $\mu_0^i$ , an observed increase in  $M_t^i$  will still lead to an greater likelihood of contraception in the succeeding period than in (i) above. This is because individuals guess that part of the increase in family size might be due to a higher fecundity level.

Also, as a theoretical note, were one to solve out the RS model, one could, in principle, get a demand function for contraception that is analogous to the EC reduced-form equation (1.2'). This would relate contraceptive use levels to exogenous things in the RS model, like market costs of contraception. It is likewise possible to get an estimate of the EC potential supply by calculating the reproduction function at each  $t$  with  $Z_t^i$  set to zero. With a few assumptions it is even possible to get an analog to the EC demand concept: assume that individuals know  $\mu_0^i$  and that there is no randomness  $\epsilon_t^i$ ; if the model is then solved for contraception levels at  $r_t = 0$  using these contraception levels to calculate  $S_t^i$  and then summing over  $t$  we obtain a corresponding concept to the "demand" for children in EC.

### 2.2.2. Empirical Implementation.

As indicated, RS do not undertake the unenviably difficult task of solving out the dynamic program for a set of jointly estimable fertility demand and supply equations. They do, instead, adopt the compromise strategy of estimating the reproduction equation (2.3) (which has characteristics of a fertility supply equation), reserving (2.1) and (2.2) as background considerations for the selection of instruments and interpretation of results.

The unfortunate weakness of this strategy is that all of the interesting dynamics of the theoretical model are lost due to the time-aggregation procedure that gets them an estimating equation; the version of (2.3) that gets to be estimated below does not feature any dynamic simultaneity or lagged adjustment in either the structural equation or the instrumental-variables equation of their two-stage least squares procedure. Consequently, like the EC model, the empirical expression of the RS model is also properly regarded as a static lifetime choice specification.

The mechanics of their empirical study are as follows:

- I. Take (2.3) and divide by the number of time periods  $T$  to get an "average fertility" equation:

$$(2.3') \quad \bar{S}^i = \mu_0^i + \mu_1 \bar{a}^i + \mu_2 \bar{Z}^i + \bar{\epsilon}^i$$

where  $\bar{S}^i$  is now the average fertility over  $T$  periods of the  $i$ th individual (i.e., the birth rate); and  $\bar{Z}^i$  is the proportion of  $T$  periods that contraception was in use. Under the presumption that the analyst does not know  $\mu_0^i$ , estimation of (2.3') by OLS is biased and inconsistent, because the explanatory variable  $\bar{Z}^i$  is correlated with the joint error term  $\mu_0^i + \bar{\epsilon}^i$ . Correlation with  $\mu_0^i$  follows from results (ii)–(iii) above: more fecund individuals tend to self-select into more rigorous contraceptive regimes. Correlation with  $\bar{\epsilon}^i$  proceeds from the fact that the realizations  $\epsilon_t^i, \epsilon_{t-1}^i, \epsilon_{t-2}^i$ , etc., which are components of the mean  $\bar{\epsilon}^i$ , are among the information variables that the individual uses to form her choice of  $Z_t^i, Z_{t+1}^i, Z_{t+2}^i, \dots$ , which in turn are components of  $\bar{Z}^i$ . This requires an instrumental-variables procedure like that adopted in EC.

- II. The authors argue that "since  $F$  [i.e.,  $\bar{Z}^i$  above] reflects the couple's demand for children, it is a function of preferences, prices, and [husband's] income. As long as these variables are orthogonal [i.e., uncorrelated] to fecundity, the usual set of fertility demand variables may serve as instruments for  $F$  and permit identification of the supply technology." (Rosenzweig and Schultz, 1985, p.998.) Hence, step II consists of estimating an equation like the following by

OLS or a Tobit regression (one would favor a Tobit procedure if a large enough proportion of sample data on  $Z^i$  were zero):

$$(2.4) \quad Z^i = \varphi_0 + \varphi \cdot \bar{x}^i + \zeta^i$$

where  $\zeta^i$  is a random error and  $\bar{x}^i$  are averages of "the usual set of fertility demand variables," (e.g., personal characteristics, market goods prices, husband's income, etc. See RS (1985) for a detailed list of these instruments.)

III. Using now the estimates of  $\varphi_0$  and  $\varphi$  obtained in (II) construct predicted values of  $\hat{Z}^i$  from the sample of  $\bar{x}^i$  and use these predicted values in place of  $Z^i$  in (2.3'). Estimate (2.4) by OLS estimation of (2.3') with  $\mu_0^i + \bar{\epsilon}^i$  as the joint error term. This will supply us with consistent estimates for  $\mu_1$  and  $\mu_2$ .

IV. The remaining problem is to obtain a consistent estimate of  $\mu_0^i$ . This is easy if we understand that the mean error  $\bar{\epsilon}^i$  in (2.3') is zero for large T. Hence, the difference between  $S^i$  and  $\hat{\mu}_1 \bar{a}^i + \hat{\mu}_2 \hat{Z}^i$  ( $\hat{\mu}_1$  and  $\hat{\mu}_2$  are the estimates obtained from step III) is a consistent estimate  $\hat{\mu}_0$  of  $\mu_0$ .

### 2.2.3. Evaluation of the RS Model.

Several remarks are in order; we will first consider some technical comments on the RS empirical strategy and then turn to the larger issue of comparing the theoretical model of RS with that of EC.

We have already noted above that the time-aggregation employed in the RS estimation strategy identifies and allows for consistent estimation of the reproduction equation parameters  $\mu_0^i$ ,  $\mu_1$ , and  $\mu_2$ , but the price of this is that aggregation reduces the empirical model to one of static lifetime choice behavior. Consequently, one cannot ask of equations (2.3') and (2.4) questions like "How will greater access to contraception affect the timing or spacing of births and the age distribution of the future population?" or "Does birth spacing increase with parity?"

Secondly, with respect to the empirical implementation discussed in I-III above, the

first obvious question that one has is how reasonable are the "usual set of fertility demand variables" that RS select in terms of their orthogonality to the combined error term  $\mu_0^i - \epsilon^i$ ? While there are theoretical grounds for such orthogonality to actually be the case, it would have been reassuring in their 1985 study if RS had reported some test against this null hypothesis applied to (2.3').<sup>9</sup>

Lastly, observe that the relationship between the empirical equations that RS actually estimate and the underlying dynamic program is not all that tight. Instead of solving the model, or some specific versions of it, they go ahead and directly estimate the reduced-form for contraceptive use (2.4) and then the time-aggregated reproduction equation (2.3'). It is an open question therefore as to how seriously misspecified is the estimation procedure based only on equations (2.4) and (2.3').

Consider the original dynamic programming problem (2.1)–(2.3). Under the original theory it not unreasonable to suppose that the reduced-form (2.4) is a function also of individual preference shocks  $\Theta^i$ . What happens if some of these time-invariant preference shocks are also unobservable like the time invariant fecundity shock  $\mu_0^i$ ? One would have, instead, a joint shock  $\Theta^i + \mu_0^i$  which remains unobservable to the analyst. There is still hope for consistent estimates of the reproduction function parameters  $\mu_1$  and  $\mu_2$  as long as "the usual set of fertility demand variables" that are used as instruments  $\bar{x}^i$  in (2.4) remain uncorrelated (orthogonal) to this joint shock. Unfortunately, this may not be true for the set of personal characteristics that are typically included in the RS instrumental-variable regression (2.4). Further, even if these variables are dropped so that  $\mu_1$  and  $\mu_2$  can be consistently estimated, one cannot arrive at a consistent estimate of  $\mu_0^i$  separately (or  $\Theta^i$  for that matter), only a consistent estimate of the sum  $\Theta^i + \mu_0^i$ . Thus one cannot analyze the effects of isolated changes in  $\mu_0^i$  on fertility variables or other variables of policy interest.

To handle this possibility one suggestion is to allow for an individual-varying intercept  $\varphi_0^i$  in (2.4) to capture the unobservable preference shocks  $\Theta^i$ . We would have to reformulate (2.4) as

$$Z_t^i = \varphi_0^i + \varphi x_t^i + \zeta_t^i$$

One could then obtain consistent "within" estimates of  $\varphi_0^i$  given enough observations  $t$  for



each individual  $i$ . This approach, of course, is more demanding on the data set since it requires a time series for each observed individual.

We now turn to a comparison of the theoretical RS model (2.1)–(2.3) vis-à-vis its competitor, the EC model of Section 1 above. From a theorist's viewpoint the RS model is certainly more satisfying in terms of the way economic behavior is specified. The advantage of stating the behavioral problem in an explicit form is that confusion in the interpretation of the outcomes of choice behavior is minimized. Moreover, because the estimating equations have a foundation in a rigorous theoretical structure we can be somewhat more confident about what misspecifications might be important, and which are not. The resulting equations that RS estimate are thus less sensitive to potential correlation problems between fertility control variables such as contraception and the fertility equation error, though as discussed above, they are not entirely free of them. And again, despite this the original EC study was careful to address the correlation problem. To study the effect of changes in structural variables (such as unobserved shocks to preferences), appropriate modifications of the theory and estimating models are easy to introduce since the formal structure of the model is explicit.

Another nice feature of the theoretical RS model is that the stochastic side is modelled more explicitly than in EC. One knows exactly what type of stochastic disturbances the model regards as important and hows these enter the decision-making problem of individuals making fertility choices, and how these shocks carry over into relationships derived from the basic maximization program. In the EC framework preferences (demands) and the reproduction function are regarded as subject to stochastic shocks which may or may not be uncorrelated across equations, which may or may not be correlated with regressors, etc. The payoff to using the more theoretically rigorous RS approach is that the analyst is better able to identify and handle specific nonorthogonality problems appearing in the fertility equation than would a naive application of the proximate determinants approach.

The precision with which the RS model is specified, while useful in identifying potential problems, as well as new results, may become a drawback, however, if flexibility rather than precision is important for the purposes to which the model will be used. One advantage of the EC framework over RS, apart from its simplicity of structure, is that it is more easily adapted to suit special needs—one could include more variables (like direct policy) or extend the basic model to explain other variables in terms of more fundamental causal factors, albeit in an ad hoc fashion. One need not worry about the microfoundations of behavior. The down side of this is that there are applications of a model (policy simulation is one) where knowledge of the theoretical structure of the model is valuable

and the lack of microfoundations may be damaging to policy inference.

The practical value of the RS model, in terms of explanatory power, predictive performance, and policy usefulness, is still an open question. It is still not demonstrably superior for policy studies of fertility than other approaches, notably that of EC (correctly estimated.). Since, from a historical standpoint, there has been a strong policy motivation behind this line of research, whether the RS approach gets to be broadly adopted may well lie in the hands of policy-oriented users rather than the academic consumers of the model. The other characteristic of the RS model that works against its wider usage are the data requirements for model implementation. To carry out time-aggregation in the estimation procedure requires the use of data that is both cross-sectional and time-series, as the 1970 U.S. National Fertility Survey and the 1975 resurvey which they used in their 1985 study. The RS model cannot exploit most existing data sets, which are either purely cross-sectional or purely time-series in structure.

Data requirements notwithstanding, the RS model remains a serious competitor to the more widely-used EC framework for several good reasons: its rigorous theoretical structure, its explicit treatment of the stochastic shocks on behavior, better handling of stock and flow variables as well as simultaneous relationships. Potentially, the RS theoretical framework may be further developed to admit the empirical study of lagged and simultaneous dynamic relations between fertility variables. Such a study could potentially exploit its close relationship with other stochastic dynamic models of fertility choice to provide guidance about stylized results, and to cross-check or buttress novel empirical findings. In its present form, the empirical RS model is of limited usefulness for the analysis of flow fertility variables as opposed to stock variables.

### 3. Montgomery.

In this section we review a reformulation of the EC framework that first appeared in an article by M. Montgomery (Demography, 1987). The Montgomery model, like its predecessors, falls under the category of static fertility models. Though descended from the EC model, it is a considerable revision upon the original EC model and can be regarded as an independent model in its own right.

The principal motivation behind Montgomery's reformulation is the criticism that had been levelled at the EC concept of demand for children (see Schultz's 1986 review). As has been discussed previously, the EC demand concept suffers from several problems of interpretation. Montgomery avoids all of these problems by allowing the demand for children to depend upon regulation costs in the standard way that economic demand

curves are defined. The principal advantage of this reformulation is that by redefining the concept of demand for fertility in the standard way, one can then access the large body of standard (hence, less controversial) analytical techniques for estimating demand systems that had previously gone unexploited. The cost of this reformulation is that one distinguishing feature of the EC framework, its one-way causal relationships, is no longer preserved.

### 3.1. Theory.

Imagine, as in the EC model, that individuals make a *lifetime* decision as to how many surviving children they would like to have, together with how much of market goods they would like to consume. Now, however, they must also choose these jointly with the level of contraceptive effort required to realize their desired family size. Formally, assume individuals maximize a utility function that depends on lifetime consumption  $C$ , family size,  $S$ , and "contraceptive effort,"  $Z$ :

$$(3.1) \quad U(C,S,Z) = C^{\alpha_0} \cdot S^{\alpha_1} \cdot Z^{\alpha_2} ; \text{ with } \alpha_0, \alpha_1 > 0 \text{ and } \alpha_2 < 0.$$

In Montgomery's model contraceptive effort,  $Z$  is defined as

$$(3.2) \quad Z = B/N$$

where  $B$  = maximum number of expected births  
 $N$  = expected number of births

$S$  in turn, is the product of a (given) survival rate  $r$  (this is  $1-x_7$  in terms of the original EC model) and  $N$ :

$$(3.3) \quad S = rN$$

The number  $B$  is presumed given to and known by the individual but is unobservable to the analyst. Elementary substitutions allow us to write  $U$  as a function of consumption  $C$ , births  $N$ , the and the fixed numbers  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $B$ , and  $r$ :

$$(3.1') \quad U(C,N;B,r) = C^{\alpha_0} \cdot N^{\alpha_1 - \alpha_2} \cdot B^{\alpha_2} \cdot r^{\alpha_1}.$$

Note that for this choice of utility function, changes in maximum expected births  $B$  and in survival rates  $r$  essentially rescale individual utility levels exponentially, but don't affect the rate at which individuals trade off less  $C$  for more  $N$  to maintain welfare  $U$  at a given (fixed) level. We will have more to say on this below.

Individuals then choose  $C$  and  $N$  to maximize (3.1') subject to two constraints. There is a lifetime budget constraint

$$(3.4) \quad \begin{aligned} I &\geq pC + qS \\ &= pC + qrN \end{aligned}$$

where  $p$  and  $q$  are the lifetime costs of each unit of  $C$  and each unit of  $N$  respectively. There is also a rationing constraint which just says that expected births cannot exceed the (unobservable) maximum expected births:

$$(3.5) \quad N \leq B.$$

It will be observed that the budget constraint does not include any market costs of regulation, though it is easy enough to incorporate these. The psychic costs of regulation, however, do appear in this model via the presence of contraceptive use  $Z$  as an argument of the utility function (3.1) (and  $\alpha_2$  is negative). Although  $Z$  gets to be substituted out in (3.1'), the psychic costs continue to be manifest in the form of the parameter  $\alpha_2$ . Hence the demand for children will depend (negatively) on regulation costs. This is why the Montgomery model is a significant departure away from the EC framework.<sup>10</sup>

Although in Montgomery's model the independence of demand from causal effects emanating from regulation costs is gone, Montgomery is able to preserve the decomposition of the EC supply of births concept into proximate determinants. This observation is important as it allows Montgomery to later estimate the maximum number of expected births  $B$  as a linear sum of proximate determinants  $x$ .

Figure 2 represents this model informally, using standard indifference-curve diagrams:

In figure 2. the shaded area represents those combinations of consumption and births which are feasible given prices, income, and the biological-demographic parameters  $B$  and  $r$ .  $U_1$  and  $U_2$  are representative indifference curves of two different individuals, one of whom faces a binding rationing constraint. In the diagram individuals who are not supply-constrained (for whom  $B > N$  in the diagram) will end up choosing a level of  $N^*$  which is strictly less than the maximum level of births, as on point  $O_1$ . For such individuals the operative constraint on the number of births comes from the demand side—tending to have relatively stronger preferences for consumption over children, they voluntarily choose to restrict their births to levels lower than the natural maximum  $B$ . On the other hand, individuals with preferences like  $U_2$  would like to attain a point like  $O_1$  in the diagram where their utility level is highest subject to the budget constraint. Because, however, of the rationing constraint the best that they can do is the corner solution  $O_2$ .

For individuals represented by  $U_1$ , their choice of realized births and consumption is given by the interior solution  $O_1$ . This solution is

$$(3.6a) \quad N^* = \left[ \frac{(\alpha_1 - \alpha_2)}{(\alpha_0 + \alpha_1 - \alpha_2)} \right] \cdot \left[ \frac{I}{qr} \right]$$

$$(3.6b) \quad C^* = \left[ \frac{\alpha_0}{(\alpha_0 + \alpha_1 - \alpha_2)} \right] \cdot \left[ \frac{I}{p} \right]$$

For individuals represented by  $U_2$ , theirs is the corner solution

$$(3.7a) \quad N^* = B$$

$$(3.7b) \quad C^* = \left[ \frac{I - qrB}{p} \right]$$

Inasmuch as the sample contains individuals whose solutions are described by (3.6) and individuals described by (3.7) it will become important in the empirical implementation of this model to take this into account. Before turning to the empirical implementation and issues, however, let us make a short detour into some technical aspects of the theory that have relevance to how one might study the effects of specific types of "modernization" in the Montgomery framework.

A fair amount of the policy research that followed the lead of The Fertility Revolution was concerned with estimating the effects of various measures of modernization on births and related economic choices (refer back to Section 1.2.1, specifically equation (1.3) of the EC model.) In modelling the effects of modernization and policy, it would be convenient, where possible, to be able to isolate those effects of modernization and policy acting *solely* on the survival rate  $r$  or the maximum expected births  $B$ . This way, one might focus directly on the effect of specific measures of modernization on the supply-side structures of the budget and rationing constraints without having to be concerned about any confounding effects on preferences (the utility function) and the demand side of the fertility decision. While modernization in general does affect relative prices, preferences, and incomes, it may be reasonable that some specific types of modernization affect only  $B$  or  $r$  but not the preference parameters  $\alpha_0$ ,  $\alpha_1$ , and  $\alpha_2$ .

For example, improvements in medical procedures might lead to longer fertility lifetimes, which in turn would increase  $B$ . One could suppose that this type of modernization would not affect individual preferences between consumption and family size. In Montgomery's framework, this independence of modernization effects on  $B$  from effects on preferences is guaranteed. Looking at the utility function (3.1') shows that higher  $B$  affects the *level* of utility  $U$  by the factor of  $B$  raised to the power  $\alpha_2$ , but at any fixed level of utility like  $U_1$  or  $U_2$  in the diagram, the slope of the indifference curve  $U$  is independent of  $B$ :

$$\left. \frac{dC}{dN} \right|_{U \text{ fixed}} = - \left[ \frac{\alpha_1 - \alpha_2}{\alpha_0} \right] \cdot \left[ \frac{C}{N} \right]$$

Since changes in  $B$  do not affect the budget constraint, modernization that works to increase  $B$  will fail to change fertility outcomes once the rationing constraint fails to bind. (Note that  $B$  is nowhere to be found in equations (3.6a) and (3.6b) for optimal  $C$  and  $N$ .) In this way the Montgomery model captures very nicely the EC notion of how, as societies modernize, fertility becomes less supply-constrained and more demand-constrained. There is an interesting, and novel result here, though. Observe that as the rationing constraint becomes nonbinding, any further increases in  $B$  (due to modernization, say) *lowers* rather than raises overall utility and welfare. This is because it forces individuals to adopt more rigorous contraceptive efforts, which in turn detracts rather than adds to utility (i.e.,  $\alpha_2$  is typically negative.) This runs counter to the principle that relaxing a constraint

cannot bring welfare down, and also to the seat-of-the-pants presumption that modernization makes one better off unambiguously.

The sharp separation between supply-side effects and preference effects of changes in  $B$  is possible because of the special form of the utility function adopted by Montgomery. The Cobb–Douglas form (3.1') guarantees that the curvature of the indifference curves is not affected by changes in  $B$ . For more general specifications of  $U$  the slopes of the indifference curves may depend  $B$  so that modernization acting through  $B$  might have demand-side effects as well.

As for modernization that increases the survival rate  $r$ , in Montgomery's model this again has no effect on the slope of the indifference curves (hence, has no impact on tradeoffs on the demand side.) The difference is that unlike changes in  $B$ , changing  $r$  will have an impact on the slope of the budget constraint represented in figure 2, even if relative prices  $q/p$  and real income  $I/p$  remains unchanged. An increase in  $r$  will thus result in a smaller  $N$  with consumption  $C$  *unchanged*. The level of utility, however, *may go up or down* because  $U$  gets scaled up by the factor of the change in  $r$  raised to power  $\alpha_1$  (refer back to equation (3.1').) This offsets the drop in  $N$ . Modernization acting through changes in  $r$  thus has an ambiguous effect on welfare depending on parameters including those that reflect preferences ( $\alpha_0$ ,  $\alpha_1$ , and  $\alpha_2$ .) This is yet another new result, which hopefully might be examined empirically.

### 3.2. Estimation.

Montgomery's estimation strategy is a simple and elegant application of the technique of switching regressions. First, we can regard the sample observations of  $N$  and  $B$  as being realizations or draws from one of two regimes, the first regime being

$$(3.8) \quad N = \lambda^1 \cdot x + \xi^1$$

and the second regime being

$$(3.9) \quad N = B = \lambda^2 \cdot x + \xi^2$$

where  $x$  is a vector of exogenous variables<sup>11</sup> affecting demand, the constraint, or both of these. (Some exclusion restrictions on the parameters will be necessary to identify (3.8) from (3.9)—to be identified from each other the two equations cannot share the exact

same set of  $\mathbf{x}$  variables.)

The problem is that the analyst does not know the level  $B$ . To get around this, Montgomery employs the following additional identification criterion. If the individual ever used contraception she should be classified as a draw from the first regime. If she never used contraception, she is a draw from the second regime, since for individuals in this regime optimal family size exceeds the upper limit  $B$ , obviating the need to contracept. This classifies the sample into two subsamples, one for each regression.

One then could go ahead and estimate (3.8) and (3.9) by OLS but because of the truncated sample (regime 1 is truncated above at the level  $B$ ; besides we never observe negative values of  $N$  or  $B$  in either regime) the Tobit procedure is called for to estimate the switching regressions model (3.8)–(3.9). (Again, for details on the Tobit procedure, a good reference is Maddala, 1983.) Under this procedure the contribution to the sample likelihood of observations from regime 1 is

$$(3.10) \quad [1/\sigma^1] \cdot \phi[(N - \lambda^1 \cdot \mathbf{x})/\sigma^1]$$

where  $\sigma^1$  is the standard deviation of  $\xi^1$  and  $\phi$  the normal probability density function. The contribution to the sample likelihood from observations in regime 2 is

$$(3.11) \quad 1 - \Phi[(N - \lambda^2 \cdot \mathbf{x})/\sigma^2]$$

where  $\Phi$  is the normal cumulative density function and  $\sigma^2$  the Tobit standard error of  $\xi^2$ . A joint likelihood function can then be constructed and maximized by choice of the parameters  $(\lambda^1, \lambda^2, \sigma^1, \sigma^2)$  using the full sample classified according to (3.10) or (3.11).

The above procedure delivers consistent estimates of the model parameters, however, Montgomery notes that it may be possible to get more "efficient" (i.e., smaller variance) estimates. This proceeds from a reinterpretation of the earlier EC demand concept in the context of Montgomery's theoretical model. Suppose, as Montgomery argues, that survey responses to ideal family size questions measure the demand for births  $N$  in the hypothetical situation of no psychic regulation costs ( $\alpha_2 = 0$ ) and assured survival ( $r = 1$ ). Then this implies that we have an additional equation restricting the parameters of (3.6a) and (3.7a). Imposing this restriction in the subsequent Tobit estimations will increase the precision of the estimates, though, again, this extra restriction is not necessary for consistent estimation.



### 3.3. An Evaluation.

At the "cost" of introducing simultaneity into the model (which some might even consider a desirable thing) Montgomery is able to do away with much of the confusion surrounding the demand concept of Easterlin and Crimmins without losing any other useful features of the original EC model. Adopting the Montgomery reformulation allows one to apply tried and tested approaches to demand estimation. In addition, the Montgomery reformulation also makes some novel predictions of its own about how modernization acting through supply-side channels affect welfare. We regard the Montgomery model, therefore, as a separate important alternative to modelling lifetime fertility decisions. The other significant advantage of the Montgomery model is that it makes no more requirements on the data than those essentially imposed by the EC framework. Data sets for implementing this model abound.

The principal limitation of Montgomery's model is that, like the EC and RS models, it is a static lifetime fertility choice model. The principal variable of that all these static models seek to explain is total births over the individual's life-cycle; one still cannot properly analyze questions of birth timing and birth spacing or intertemporal tradeoffs between children and other goods. To address these questions requires the use of a model that generates estimating equations which are dynamic.

#### *Dynamic Models of Fertility Choice.*

We have noted that all three previous models are essentially models that have been used to explore the economics of lifetime fertility. Recent research on the fertility decision, however, has been oriented more towards the analysis of the spacing decision over time and under uncertainty. While this reflects, in part, the same trend in other areas of economics, it is also a conscious attempt on the part of researchers to overcome many of the limitations of static models. Static models do not capture the sequential nature of the fertility decision. Static models cannot adequately address questions about how changes in the *time profile* of costs of contraception, wages, incomes, mother's education, or mortality risks affect either completed family size or the spacing of children. Static models which assign consumption value to children also neglect the durable and irreversible characteristic of children as consumption goods. At the same time there exist stylized empirical regularities which have dynamic characteristics. Examples are the apparent convergence of fertility rates in developing countries (World Development

Report, 1984), the tendency of births to be more narrowly-spaced among higher fertility women, the tendency for women to space births as "parity" (i.e., attained fertility level) increases, the finding that current and future levels of the woman's wage and husband's income are important in explaining the number of births and the timing of the first birth (Heckman and Walker, 1990), the observation that recent U.S. fertility levels appear to be countercyclical to the business cycle (Mocan, 1989), etc.

Finally, the need to introduce more general and "realistic" types of uncertainty reflecting patterns of serial correlation and correlations with time-varying levels of economic or biological variables require the use of an explicitly dynamic framework.

In this section we review three such dynamic models each of which emphasizes different aspects of the dynamic fertility decision. These are, respectively, the model of Wolpin (1984), Newman (1988), and Barro and Becker (1988, 1989). Given that this research program is in relative infancy, it is difficult to tell which of these models will prove to be seminal for the empirical end of future research. While the underlying theoretical framework is very similar for this class of models, it has been observed that when it comes to estimation there has not appeared a consensus about the appropriate empirical model to use. In this review, therefore, we will provide more details on the theoretical structure of the class of dynamic stochastic models and the predicted theoretical relationships, although we do discuss various estimation strategies that have been suggested by this literature. Readers interested mainly in the details of econometric specifications and empirical results of these dynamic models should consult the original references to supplement the discussion below.

#### 4. Wolpin.

##### 4.1. Model.

One of the seminal papers in the dynamic approach to fertility choice is Wolpin (1984). Since his model contains several features that exemplify a distinct approach to the modelling of fertility dynamics, we examine it in some detail.

Like other dynamic models that are reviewed here Wolpin sets up an intertemporal maximization problem which is truly dynamic in the sense that present decisions are forward-looking and past outcomes affect future outcomes. His model is set up in discrete time periods, with each period being measured as that length of time over which a birth can occur. In each period of her lifetime the individual receives utility from consumption of market goods in the period and the existing stock of children for the period only. Let

the period utility function be  $U(C_t, M_t)$  where  $C_t$  is current consumption and  $M_t$  the stock of children.

The model accommodates three types of uncertainty, a random shock to the individual's utility function (which reflects changes in tastes over time), randomness about the occurrence of infant mortality in every period of life, and randomness in current income. Births are assumed to occur only in the fertile stage of life, however the duration of this is known with certainty to be  $T^f$  periods. This last assumption is made largely for tractability reasons.

Uncertainty makes it such that at any instant in her lifetime the individual knows current and past outcomes but can only form guesses about the likely future values of relevant variables. Given uncertainty, at the onset of her fertile lifetime ( $t=0$ ) the individual is taken to maximize the *expected value of the discounted sum* of period utility over her lifetime  $t=0, \dots, T$  (including infertile periods):

$$(4.1) \quad E_0 \sum_{t=0}^T \delta^t \cdot U(C_t, M_t, \xi_t)$$

where  $\delta$  is the individual's subjective discount factor and  $E_0 x_t$  represents the rational expectation at time 0 (i.e., the true statistical expectation of  $x_t$  conditioned on information available in the first period of the decision problem.) The period utility function has been extended to include random influences  $\xi_t$ . In his original model Wolpin posits a quadratic utility function of a specific type:

$$(4.2) \quad U(C_t, M_t, \xi_t) = (\alpha_1 + \xi_t)M_t - \alpha_2 M_t^2 + \beta_1 X_t + \beta_2 X_t^2 + \gamma M_t X_t.$$

We will not dwell too much on the implications of choosing this particular functional form over other possible forms. We note only that (i) a quadratic form for period utility implies that downside and upside risks to  $C_t$  and  $M_t$  are weighted equally, and that (ii) the inclusion of the shock  $\xi_t$  to preferences for children matters later on for the estimation.

The second type of uncertainty involves the probability of infant mortality. Wolpin assumes that the stock of children  $M_t$  evolves according to

$$(4.3) \quad M_t = M_{t-1} + n_t - d_t$$

where the birth process  $n_t$  equals 1 if a birth occurs in the period, and zero otherwise; the death process  $d_t$  equals 1 if the child born in period  $t$  dies, and zero if the child goes on to

live to the next period. For simplicity Wolpin assumes that children live forever (i.e., outlive their parents) if they survive the first period of life, citing evidence that most of child mortality occurs in the first few years of life. Death occurs if the value of an indicator function falls below zero:

$$(4.4) \quad d_t \begin{cases} = 0 \text{ iff } G_t \pi + u_t \geq 0 \\ = 1 \text{ iff } G_t \pi + u_t < 0 \end{cases}$$

where  $G_t$  is a vector of variables related to the likelihood of a death occurring (examples might be household characteristics, community variables, policy variables, biological determinants, and time itself.) The randomness of death comes from the inclusion of a random term  $u_t$ , which is assumed independent and identically distributed.

The third type of uncertainty involves shocks to household income. These shocks affect the sequence of budget constraints (one for each period) that the individual faces:

$$(4.4) \quad C_t + b(n_t - d_t) + cn_t = I_t$$

where  $b$  is the per unit maintenance cost of surviving children,  $c$  is the cost per unit of a birth in period  $t$ , and  $I_t$  is household real income, measured in terms of the consumption good. (Observe that the prices  $b$  and  $c$  are assumed constant over time. Note as well that that the maintenance costs are assumed to be expended on the surviving children for the period  $t$  only, not for the entire stock of children alive in period  $t$ . Finally note also that there is no saving or wealth accumulation in this model; the only means of transforming income today into future utility is through having children as the stock of children  $M_t$  appears in the period utility function for future periods.) Income is assumed random, obeying the process

$$(4.5) \quad I_t = H_t \psi + v_t$$

where  $H_t$  are exogenous determinants of income (e.g., age) and  $v_t$  is an independent, identically distributed random variable with zero mean and finite variance.

The basic problem of the Wolpin model may now be stated: the individual chooses a sequence of births  $\{n_t\}_{t=0,\dots,T}$  and consumptions  $\{C_t\}_{t=0,\dots,T}$  to maximize (4.1) subject to (4.2)–(4.5).

#### **4.2. Discussion.**

What are the distinguishing features of Wolpin's theoretical model that make it representative of a particular approach to fertility dynamics? The important ones are:

- i. The model assumes that individuals have perfect control over their fertility levels, hence the occurrence of a birth  $n_t$  is regarded as a variable that the individual can choose directly.
- ii. In relation to (i), there are no explicit costs of regulating births, only costs incurred if a birth occurs.
- iii. Individuals have no control over the death process or the interval over which a child is exposed to the risk of death.
- iv. There is no capital or wealth accumulation. Intertemporal substitution is carried out by having more children.
- v. Uncertainty enters by way of randomness in income levels and shocks to preferences over family size.

Less important distinguishing features of the model are that:

- vi. Child births and deaths in each period are modelled as zero-one processes.
- vii. The duration of the fertile period of life  $T^f$  is known with certainty.
- viii. Maintenance costs apply only to new additions to the flow of surviving children in the period,  $n_t - d_t$ , not to the entire stock of children,  $M_t$ .
- ix. Prices are assumed steady over the life cycle.
- x. The model is set up in discrete time.

These last four features are less important as distinguishing characteristics, since they can be changed without affecting the main conclusions of the Wolpin model. According to Wolpin, the main analytical results are that:

- i. The model can generate a wide variety of timing and spacing patterns for births.
- ii. The model can generate a wide variety of replacement patterns for children.
- iii. It is (unfortunately) not generally the case that high fertility incidence is associated with regimes of high infant mortality.

The last result is a consequence of the stylized assumption that a child is assured survival for future periods if it survives the first life period. Under this assumption it will be typical for optimizing individuals to raise fertility only after a death has occurred. It is somewhat unfortunate in the sense that most evidence from developing areas appear to indicate that high infant mortality is positively associated with higher fertility levels.

#### 4.3. Estimation.

Estimating a dynamic programming model generally requires the adoption of one of several strategies: one can either (a) estimate general structural or reduced-form relations which are indirectly related to the theoretical model, (b) solve the dynamic programming model for the exact closed-form expressions of the structural relations (i.e., the function describing optimal choice of births and consumption as a function of structural parameters such as the utility function parameters, the parameters of the death and income processes) and then estimate these closed-form expressions, (c) should closed-form expressions for the individual's decision rules be unavailable carry out an iterative numerical estimation procedure. Rosenzweig and Schultz, in the model reviewed in Section 2 above, adopted strategy (a) although they did so in a way that made the estimating model nondynamic in structure. Newman (1989), in the model we review next, is able to obtain closed-form solutions that are potentially estimable and retain the dynamic character of the model. In Wolpin's model deriving closed-form expressions for lifetimes longer than two-periods is extremely difficult, if not impossible, hence Wolpin's study employs strategy (c).

Wolpin's estimation procedure is itself simple in principle, but the mechanics of deriving and calculating the mathematical expressions involved can get extremely

complicated very quickly. Rather than go through all the derivations here, we just present the core of the procedure and refer the reader to the original source for details.

Wolpin sets out to estimate the preference parameters  $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma$  the price parameters  $b, c$  and the discount rate  $\beta$ . (In his actual empirical specification he generalizes the functional forms for utility and costs and thus adds some extra parameters.) In addition, one would like estimates of the other parameters which appear in the decision rule that the individual uses in determining whether or not a birth should occur.

Estimation is based on the following relation: let  $LU_t$  be the value of lifetime utility from period  $t$  onwards, let  $\xi_t$  continue to denote the preference shock, and let  $P_t$  be the probability at  $t$  that an infant will survive into the next period. Given the quadratic form of utility (which essentially weights upside and downside risks symmetrically in the utility sense), Wolpin now argues that whether it is optimal to have a birth in the period or not depends upon the indicator function:

$$(4.6) \quad J_t = E_t[LU_t | n_t=1, \xi_t=0] - E_t[LU_t | n_t=0, \xi_t=0] + P_t \xi_t$$

The first term on the right-hand side is the expected value at time  $t$  of future lifetime utility conditional on the individual having a birth in the current period and no shocks to their preferences for more infants. The second term on the right-hand side is the expected value at  $t$  of future lifetime utility conditional on there being no current births and no preference shocks. The third term measures the expected value of the change in utility due to any change in the individual's preferences for more children. The optimal decision rule for births is therefore:

$$(4.7) \quad n_t^* \begin{cases} = 0 \text{ iff } J_t \leq 0 \\ = 1 \text{ iff } J_t > 0 \end{cases}$$

Optimal births  $n_t^*$ , then, depend on the value of the function  $J_t$ , which in turn depends on the parameters of future lifetime utility  $LU_t$ , the parameters of the conditional distribution function that the individual uses to get the conditional expectation  $E_t$ , the survival probability  $P_t$  and the preference shock  $\xi_t$ .

If one examines (4.7) and (4.6) carefully, it will be observed that the two equations look like a classical probit model for  $n_t^*$  (see Maddala, 1983), except that one does not have a closed-form expression for  $E_t[LU_t | n_t=1, \xi_t=0] - E_t[LU_t | n_t=0, \xi_t=0]$  in terms of the

model's parameters. Is this model estimable anyway? The answer is yes, with a few additional assumptions.

Looking at (4.6), Wolpin argues that there will always be a unique value of  $\xi_t$ , call this critical value  $\xi_t^*$ , for which the indicator function  $J_t$  will be zero. ( Simple algebra says this value is just

$$(4.8) \quad \xi_t^* = \frac{E_t[LU_t | n_t=0, \xi_t=0] - E_t[LU_t | n_t=1, \xi_t=0]}{P_t}.)$$

Assume that individuals know the random shock  $\xi_t$  at the time of the current period fertility decision, but the analyst does not ever observe  $\xi_t$ . The analyst cannot then observe  $J_t$  directly, however she can still observe  $n_t^*$ . To proceed with the estimation, make the classical probit assumption that  $\xi_t^*$  is a normally distributed random variable. With this assumption the probability that the individual will have a child in period  $t$  (conditional on her existing stock of children) is given by

$$\Pr[n_t=1 | M_{t-1}] = 1 - \Phi\left[\frac{\xi_t^*}{\sigma}\right]$$

where  $\sigma$  is the standard error of  $\xi_t^*$  and  $\Phi(x)$  is the value at  $x$  of the standard normal cumulative density function. Again following the classical probit approach, the probability of no birth at  $t$  (conditional on the stock of children) is given by

$$\Pr[n_t=0 | M_{t-1}] = \Phi\left[\frac{\xi_t^*}{\sigma}\right]$$

Now classify time periods into two observable sets—the set  $\Omega$ , consisting of those periods where there is a birth, and  $\Omega^c$  which are those periods where there is no birth. For an individual  $i$ , then, the likelihood  $L^i$  of any particular birth pattern is

$$L^i = \prod_{t \in \Omega} \Pr[n_t=1 | M_{t-1}] \cdot \prod_{t \in \Omega^c} \Pr[n_t=0 | M_{t-1}]$$

where we have dropped the subscripts  $i$  on  $\Omega$ ,  $\Omega^c$ ,  $n_t$ , and  $M_{t-1}$  to conserve on notation. (Note that  $L^i$ , in general, is ultimately a function of the individual's basic parameters



$(\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma, b, c, \delta, \sigma)$  through its dependence on  $\xi_t^*$ .) For a sample of  $i = 1, \dots, I$  individuals, the likelihood function is

$$L = \prod_{i=1}^I L^i.$$

(which again depends on parameters through its dependence on  $\xi_t^*$ .)

The procedure that Wolpin employs is to search numerically for the parameter values  $(\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma, b, c, \delta, \sigma)$  that give the unique value of  $\xi_t^*$  that maximizes  $L$  (or its log.) This sounds straightforward, but the computational burden here is that at every step of the search one is perturbing the parameters to their new values to get a new value of  $\xi_t^*$ ; in order to go, however, from any given value of the parameters  $(\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma, b, c, \delta, \sigma)$  to the unique  $\xi_t^*$  one has to calculate, by (4.8), the expression

$$E_t[LU_t | n_t=1, \xi_t=0] - E_t[LU_t | n_t=0, \xi_t=0]$$

given the new parameter values and the sample data. As intimated above, this expression does not have a simple closed-form solution. In fact, to get this difference one actually has to solve the dynamic programming problem (4.1)–(4.5) for the maximal value of  $LU_t$  backwards from the ending period  $T$  all the way to period  $0$ . So that at every step of the way to the likelihood maximizing parameter values one must numerically solve a  $T$ -period dynamic program. The computational complexity of the Wolpin procedure is directly related to how many steps one has to take to find the maximum of  $L$  and the complexity of the dynamic program that has to be solved at every step. Even for simple specifications of his model, this seems to require substantial computing time, to think nothing of the demands on the analyst's programming time and diagnostic skills!

In all fairness, while the complexity of the procedure may discourage the wider adoption of his model, Wolpin claims that in order to replicate all the possible birth patterns that his model can generate, the use of an alternative structural econometric model would require the estimation of around 400 parameters, instead of the 13 that he estimates in his study. While we have no reason to doubt this claim, a more reasonable approach one could take would be to rule out in the theory some of less likely birth patterns that could appear in the data. After all, the Wolpin model does not have to explain every conceivable pattern of births to be usefully applied to most datasets.

We close this section by enumerating his empirical findings on the sensitivity of fertility to changes in various exogenous determinants:

#### 4.4. Evaluation.

To fully appreciate the significance of Wolpin's model, one need only remember that prior to his 1984 paper, economists did not have a theory of fertility whose dynamic relations were demonstrably estimable. Wolpin's model represents the first successful attempt to estimate the structural parameters of a fully-articulated theoretical dynamic programming problem. This represents a not insignificant breakthrough for the dynamic fertility choice research program in economics. The tight connection with the underlying theory allows, potentially, the analyst to conduct policy analysis that would be unavailable if one used an astructural model. For instance, a direct comparison of the welfare effects of certain policies are possible, since we possess estimates of underlying utility function parameters and the subjective discount rate. For ease of use and analysis, closed-form solutions for the structural relations are generally desirable, but are not always available. Unfortunately, for the original Wolpin model closed-form solutions are unavailable, and the complexity of the estimation procedure goes up substantially, increasing the costs of using the model. Our opinion is that it is this drawback which has kept the Wolpin model from gaining wider acceptance, and not so much for reasons having to do with the soundness of the basic approach.

The empirical relevance of the Wolpin model is evident in the econometric study in his 1984 paper. The predicted values for survival probabilities and conditional survival probabilities coming out of the model indicate that the relatively simple structure of the dynamic programming model (4.1)–(4.5) is capable of replicating some fairly complex properties of the data. The parameter estimates for the period utility function and the cost functions appear to be quite reasonable, and several specification tests indicate that the model does explain much more than just by pure chance.

#### 5. Newman.

The dynamic theory of Wolpin models individuals as having perfect control over their fertility in any period. This approach is useful for modelling what influences an individual's time-pattern of demand for births, but abstracts from any of the interesting supply-side influences such as biology or contraceptive behavior.

There is an earlier tradition of dynamic modelling which emphasizes these supply-side influences (Heckman and Willis (1975) and Hotz and Miller (1986)) by positing that individuals have indirect control over the number of births in a period, but

that such control is imperfect, subject to random influences, biological and otherwise.<sup>12</sup> In this section we examine the model of Newman (1988). It is a recent treatment in this tradition, but is unique in several important respects and worthy of detailed study. Unlike previous dynamic models within this tradition or models of the Wolpin type, Newman's model delivers closed-form solutions for the fertility problem. As we have seen in the discussion of the empirical Wolpin model, this feature could be potentially advantageous from an application standpoint. Secondly, unlike previous models of imperfect control, Newman's model allows for a continuous rather than discrete choice of contraceptive effectiveness, hence is potentially better able to characterize contraceptive efficiency, as this is typically a continuous variable. Lastly, Newman's model does introduce contraceptive costs explicitly, which is valuable in that most policy analysis regards costs as an important determinant of fertility supply; thus far, however, existing research has relied on static models to provide insight into the effects of contraceptive costs on fertility.

### 5.1. Model.

Unlike most other models of fertility choice, Newman's model is cast in continuous time. By itself this should not make too much difference in terms of the basic results. The advantage of a continuous time model is that certain birth and death processes are easier to analyze in continuous time.

Let period utility be represented by the function  $U(C_t, M_t, u_t)$  where  $C_t$  and  $M_t$  are consumption and family size, as before, and now the term  $u_t$  measures contraceptive efficiency. A period is now taken to be an arbitrarily small instant of time. Let period 0 be the onset of fertility and let  $T$  now represent the end of the fertile cycle. Let  $N_T$  represent the *stock* of consumption from period  $T$  onwards and let  $\rho$  be the instantaneous discount rate. Newman now models the individual as choosing a sequence of consumptions  $C_t$ , and contraception levels  $u_t$  as to maximize the expected lifetime welfare function

$$(5.1) \quad E_0 \int_0^T e^{-\rho t} U(C_t, M_t, u_t) dt + S(C_T, N_T, T)$$

where  $S$  is the *terminal value function* representing post-childbearing utility.

Choice is subject to a budget constraint:

$$(5.2) \quad C_t + bM_t + ru_t = I_t$$

where  $r$  is the cost per unit of increasing contraceptive efficiency; and subject to the stochastic process describing births:

$$(5.3) \quad \Delta M_t = \Delta n_t + \Delta d_t.$$

$\Delta n_t$  is a Poisson process for births with parameter  $(h - u_t)$ , i.e.,

$$\begin{aligned} \text{The probability of a birth in the interval } \Delta t \\ &= \Pr(n_{t+\Delta t} - n_t = 1) \\ &= (h - u_t)\Delta t + o(\Delta t) \end{aligned}$$

where  $o(\Delta t)$  represent terms which are smaller than  $\Delta t$ , so that  $o(\Delta t)/\Delta t \rightarrow 0$  as  $\Delta t \rightarrow 0$ . In addition,  $\Delta d_t$  is a simple death process, i.e.,

$$\begin{aligned} \text{The probability of a death in the interval } \Delta t \\ &= \Pr(n_{t+\Delta t} - n_t = -1) \\ &= M_t \mu \Delta t + o(\Delta t) \end{aligned}$$

The probability of more than one event occurring is taken to be

$$o(\Delta t)$$

and the probability of no change is

$$1 - [(h - u_t) + M_t \mu] \Delta t + o(\Delta t).$$

Finally it is assumed that  $M_0 = 0$  and that  $0 < u_t < h$ . The parameter  $h$  represents the maximum level of fecundity, and that for biological or sociological reasons, the highest level of  $u_t$  will always fall short of eliminating all fecundity.

## 5.2. Discussion.

The important features of the Newman model are:

- i. Individuals employ contraception to regulate their fertility levels.

- ii. Contraceptive effectiveness is not perfect, hence the occurrence of a birth  $n_t$  remains a stochastic process, potentially (but not modelled as) influenced by biological and socio-cultural factors.
- iii. The only sources of uncertainty are the birth and death processes. The birth process, notably, is Poisson, which is a better description of the waiting-time characteristic of the birth process than the typical independent normal.
- iv. Survival probabilities continue to be exogenous. Individuals still do not control the death process through maternal/child health inputs.
- v. Important biological and socioeconomic determinants of fertility can be accommodated into the model by appropriate modification of the birth process.
- vi. There are explicit and implicit psychic costs associated with fertility regulation, and explicit costs in each period to increasing family size. These costs per unit are treated as steady over time.
- vii. Maintenance costs apply to the entire stock of children  $M_t$ .
- viii. There is no capital or wealth accumulation.
- ix. The model is set up in continuous time.

### 5.3. Solution.

Solving the above dynamic optimization problem is essentially an application of the Bellman optimality principle. Consideration of all the details of the procedure is beyond the scope of this paper, however we can outline the basic approach and go to the bottom line. Define the *value function at period  $t$*  to be

$$(5.4) \quad J_t(C_t, M_t) = \max_{u_t} E_t \int_t^T e^{-\rho t} U(C_t, M_t, u_t) dt + S(C_T, N_T, T)$$

subject to the constraints of the problem.

The Bellman principle of optimality says that the dynamic program of period T can be split up into a backwards recursive sequence of smaller dynamic programs of T-1, T-2,...,1 periods. Solving the T-period program is equivalent to solving the sequence of T smaller dynamic programs. This principle allows us to cascade the maximization operator as to rewrite (5.4) as (ignoring the dependence of J on  $C_t$  to conserve notation)

$$\begin{aligned}
 J_t(M_t) &= \max_{u_t} E_t \left\{ \int_t^{t+\Delta t} e^{-\rho t} U(C_t, M_t, u_t) dt \right. \\
 &\quad \left. + \max_{u_{t+\Delta t}} E_t \int_{t+\Delta t}^T e^{-\rho t} U(C_t, M_t, u_t) dt + S(C_T, N_T, T) \right\} \\
 &\cong \max_{u_t} E_t \left\{ e^{-\rho t} U(C_t, M_t, u_t) \Delta t \right. \\
 &\quad \left. + \max_{u_{t+\Delta t}} E_t \int_{t+\Delta t}^T e^{-\rho t} U(C_t, M_t, u_t) dt + S(C_T, N_T, T) \right\}
 \end{aligned}$$

Since the second term on the right-hand side is  $J_{t+\Delta t}(M_{t+\Delta t})$  we get

$$\begin{aligned}
 (5.4') \quad J_t(M_t) &= \max_{u_t} \{ E_t e^{-\rho t} U(C_t, M_t, u_t) \Delta t + J_{t+\Delta t}(M_{t+\Delta t}) \} \\
 &\cong \max_{u_t} \{ e^{-\rho t} U(C_t, M_t, u_t) \Delta t + E_t J_{t+\Delta t}(M_{t+\Delta t}) \}
 \end{aligned}$$

This is Bellman's equation for the problem. In principle, this is all that is needed (together with the constraints and the initial condition  $M_0 = 0$ ) to solve the dynamic problem *numerically* up to an arbitrarily small approximation error. However, Newman seeks an exact closed-form solution for contraceptive use  $u_t$ . So more work is needed.

If we subtract  $J_t$  from both sides, divide by  $\Delta t$ , and let  $\Delta t$  go to zero, we get

$$0 = \max_{u_t} e^{-\rho t} U(C_t, M_t, u_t) \Delta t + \max_{u_t} \lim_{\Delta t \rightarrow 0} E_t \frac{J_{t+\Delta t}(M_{t+\Delta t}) - J_t(M_t)}{\Delta t}$$

After some more work on the second term on the right-hand-side of the above equation, Newman shows that the stochastic assumptions about the birth and death processes imply that this equation is equivalent to

$$(5.5) \quad -dJ_t/dt = \max_{u_t} e^{-\rho t} U(C_t, M_t, u_t) \Delta t \\ + \max_{u_t} \{ (h-u_t)[J_t(M_t+1)-J_t(M_t)] + \mu M_t[J_t(M_t-1)-J_t(M_t)] \}$$

Using a quadratic specification for  $U$  Newman carries out the maximizations in (5.5) subject to the budget constraint (5.2) and interiority,  $0 < u_t < h$ , to get

$$(5.6) \quad u_t^* = (1/2)(r^2 + \gamma)^{-1} [2r(I_t - C_t) - r(\theta + \psi M_t) - \delta] - (1/2)(r^2 + \gamma)^{-1} e^{\rho t} [J_t(M_t+1) - J_t(M_t)]$$

where  $\gamma$ ,  $\delta$ ,  $\theta$ , and  $\psi$  are utility function parameters. This is the equation for the optimal contraception rule, it is not quite a closed-form solution yet, as the unsolved function  $J_t$  still appears in the expression for  $u_t^*$ . Newman employs the method of undetermined coefficients, along with some subtle but intuitive reasoning, and finds that  $J_t(M_t+1) - J_t(M_t)$  equals

$$[\alpha + \beta(N_T)]e^{(4C_3D_1 - C_2 - \mu)(T-t)} + 2D_1M_t + (D_1 + D_2)$$

where  $\alpha$  and  $\beta$  are assumed parameters associated with the terminal value function  $S$ ,  $D_1$  and  $D_2$  are constants to be determined, and

$$C_2 = -r(r^2 + \gamma)^{-1}(b + 1/2\psi)$$

$$C_3 = 4(r^2 + \gamma)^{-1}.$$

How one determines the exact value of the constants  $D_1$  and  $D_2$  is discussed in the original source (Newman, 1989.) Suffice it to say that an identity between the coefficients of the terminal value function  $S$  and the value function at the terminal period  $T$ ,  $J_T$ , together with the initial condition  $M_0 = 0$  will deliver sufficient restrictions to determine these constants. The bottom line is that the optimal contraception rule  $u_t^*$  is a closed-form

function of the model parameters, real income, consumption, contraception costs, and the stock of existing children  $M_t$ . The exact specification is given by (5.6).

To solve for the exact form of the function  $J$ , one can replace  $u_t$  in Bellman's equation (5.4') with  $u_t^*$  and work backwards from  $t=T-1$  to 0, or use the method of undetermined coefficients described in Newman (1989).

The most important (*ceteris paribus*) predictions of Newman's model are the following:

- i. Optimal contraceptive efficiency levels decrease over time as time elapsed without a birth increases. This holds whether the individual would like to have a birth or not. If an individual wants to have a birth then  $u_t^*$  should fall as time periods elapse and no birth occurs. On the other hand, suppose an individual does not want a birth but is at risk for a birth, with each period elapsed without a birth, the individual does not need to factor in the exposure to risk of the period gone by, so  $u_t^*$  is still decreasing with  $t$ , given  $M_t$  is constant. Moreover, Newman shows that the *rate* of decrease of  $u_t^*$  is increasing.
- ii. Optimal contraceptive efficiency levels may or may not be increasing with the stock of children  $M_t$ . It is more likely that  $u_t^*$  is positively related to  $M_t$  if the environment is one where contraception costs ( $r$ ) are small, the discount rate  $\rho$  is small, the risk of mortality ( $\mu$ ) is small, and the cost of maintaining a larger stock of children ( $q$ ) is high. *Thus contraceptive control is more likely to be a positive function of the stock of children in developed countries and is likely a negative function of the stock of children in developing areas.*
- iii. The spacing of children is determined by the value of fecundability  $h - u_t^*$  at various points in the individual's lifetime. If  $u_t^*$  is a positive function of  $M_t$  then individuals with more children will have longer birth intervals due to the use of more effective contraceptive regimes.
- iv. The model can generate a variety of spacing patterns, including (but not exclusively) a pattern of births consistent with increasing birth intervals with parity.
- v. Increases in child mortality rates cause individuals to contracept less.



- vi. There are, potentially, threshold effects that govern the behavior of contraceptive choice in response to changes in the cost of maintaining children and real incomes.
- vii. Increases in the value of children at the end of the fertile period will generally cause individuals to reduce the efficiency of contraception.
- viii. Increases in the maximum level of fecundity  $h$  increase fecundability  $h-u_t^*$ , even if one accounts for optimal behavior  $u_t^*$ . This result, put another way, predicts that more fecund women will tend to have more births, and, because of the response of spacing to the realization of a birth, will tend to have children earlier in her fertile period.

#### 5.4. Estimation.

The Newman model, in a sense, rationalizes the earlier empirical work on fertility that employs hazard-rate modelling (Heckman and Willis, 1975; Newman and McCulloch, 1984.) The objective of this type of approach is to estimate the parameters of a fertility equation called a *hazard function*, which captures the probability of a birth in a period, given a certain length of waiting time, exogenous determinants of fertility, and individual attempts to regulate the probability of a birth. The basic strategy itself is to specify a likelihood function for births. This likelihood function will involve the birth hazard function  $h$  as a function of explanatory variables and model parameters. One then obtains consistent parameter estimates by finding those values of the parameters of  $h$  which maximize the likelihood function given sample data for the explanatory variables.

Use of this approach in conjunction with Newman's model presents some additional nontrivial estimation problems, however. The theoretical Newman model predicts a more complicated specification of the hazard function than that which previous hazard analysis employs. For example in the earlier empirical work of Newman and McCulloch (1984) the hazard function  $h_j$ , measuring the probability of the  $j$ th birth was assumed to be

$$h_{j=1} = e^{\beta_1 X_t + \alpha_{11} t + \alpha_{12} g(t-48) + \alpha_{13} g(t-144)}$$

$$h_{j \neq 1} \begin{cases} = 0 & \text{iff } t < 7 \\ = e^{\beta_2 X_t + \alpha_{21} (t-6) + \alpha_{22} g(t-18) + \alpha_{23} g(t-30)} & \text{iff } t \geq 7 \end{cases}$$

where  $X_t$  is a vector of explanatory variables (in their empirical work these were personal characteristics and regional variables) and the function  $g$  is defined by  $g(t-i) = t-i$  if  $t-i > 0$  and zero otherwise. The hazard function of the theoretical Newman model, though, is  $h - u_t^*$ ; this expression is even more complicated because of the complexity of the added function  $u_t^*$  capturing individual attempts to optimize the hazard of birth. In addition, there may appear in the hazard function unobserved heterogeneity in both tastes and fecundity across individuals. These will, unfortunately, be correlated with the choice of the control, hence complicating the process of estimating the coefficients of the hazard function consistently. Offsetting these difficulties is the benefit of having a closed-form expression for the optimal control  $u_t^*$  and the value function  $J_t$ . Thus far, these estimation issues have yet to be worked out thoroughly but for a recent treatment of the heterogeneity issue in a hazard analysis context see Heckman and Walker (1990). The parameters of the theoretical Newman model above, however, have yet to be estimated.

### 5.5. Evaluation.

As an alternative approach to the analysis of fertility behavior over time, the Newman model presents the analyst with several attractive features. It directly addresses a central policy issue, which is how contraceptive use and costs affect fertility dynamically. From a theoretical standpoint it is better designed to accommodate influences on fertility emanating from biological sources, as well as from fertility regulation behavior. The Newman model can generate a variety of birth spacing and timing patterns, including threshold effects. Some of these predicted relationships appear to reflect stylized patterns in the data. Both the stochastic process governing births and fertility regulation behavior are modelled to depend on (among other things) the time elapsed since the previous birth. This last feature is in accord with past approaches to the analysis of birth processes. In particular, the Newman model fits very handily with work based on empirical hazard functions, and has the added benefit of an available companion econometric methodology.

The Newman model is not, we believe, especially restrictive in its formulation. Perhaps the only potential limitation is that the continuous-time framework may preclude consideration of other useful discrete-time processes, or if not this, may create issues of time-aggregation in certain empirical contexts. These difficulties, however, have not yet manifested themselves in practical applications of hazard-rate analysis and may well be only marginally important.

## 6. Barro and Becker.

The last prototypical model we review is the model of Barro and Becker (1988, 1989) (abbreviated as BB hereafter) which, unlike all of the previous models considered, was designed primarily to look at the behavior of *aggregate fertility, in relation to other variables of primarily macroeconomic interest*. The motivation for this model is partly the resurgent interest in models of economic growth in which some of the sources of growth are the consequence of endogenously made economic decisions. To illustrate the macroeconomist's perspective on fertility, consider the well-known Solow-Swan growth model. That model holds that maximizing steady-state per-capita consumption in each period requires obedience of per-capita capital  $k_t$  to the "golden rule":

$$f'(k_t) = \lambda_t + \delta_t$$

where  $f'(k_t)$  is the marginal productivity of the last unit of installed effective capital,  $\delta_t$  is the rate of depreciation of capital in the period, and  $\lambda_t$  is the growth rate of the labor force. As a second example, consider the Cass-Koopmans growth model with discounting and variable savings rates. There lifetime utility is maximized when

$$f'(k_t) = \lambda_t + \delta_t + \beta(1+\lambda_t)$$

where  $\beta$  is the discount factor. In both cases  $\lambda_t$  figures prominently in the macroeconomic optimality condition for capital accumulation.

Since the growth rate of the labor force is directly connected to the growth rate of the population in question, macroeconomists are extremely interested in estimates and determinants of population growth. In the older tradition of macroeconomic growth models,  $\lambda_t$  was often treated as exogenous to the growth process. Since the ability to control fertility has become more relevant in recent decades,  $\lambda_t$  has become more influenced by conscious economic choice. Especially salient, then, to the current research program of macroeconomics is the issue of whether *endogenous* fertility decisions are a potential source of *sustained* growth. A second important related question is that of how fertility decisions are related with the decision to invest in children's human capital and quality.

The issue of the relationship between investment in children and the demand for, or realized number of children, is a fairly traditional. The pioneering work in this area relied on static models of fertility vs. consumption choice emphasized in Becker (1960) and

Becker and Lewis (1973). Back then, however, many interesting questions of the quantity-quality tradeoff awaited the application of dynamic techniques to the microeconomic choice problem. Moreover, the task of integrating the microeconomics of household fertility and child investment decisions into a macromodel awaited a time when the microfoundations of macroeconomic models were better understood. This did not happen until after the older Keynesian structural macromodels gave way to representative agent models of the new classical type in which the introduction of fertility choice into a complete structural model would be both natural and straightforward. The culmination of efforts to relate fertility to the macroeconomy is the model of Barro and Becker (1988, 1989.) The contribution of these authors is a completely specified model of a (closed) macroeconomy where endogenous fertility decisions are integrated into decisions on the optimal time profile of per-capita consumption and additions to the capital stock.

### 6.1. Model.

In the sequel, we follow the development of the BB model given in their 1989 Econometrica paper. As we will probe the technical features of the model in the subsequent discussion, readers interested in the basic structure and the results should read omit the last half of this section as well as the next section and go directly to section 6.3 on the analytical results.

Unlike the previous Wolpin and Newman models which look at household fertility decisions over the woman's life-cycle, the BB model is an model of *aggregate lifetime fertility decisions over many generations*; it resembles, however, the framework of the other dynamic models of household fertility in that a continuing sequence of generations may be modelled as behaving like an infinitely-lived individual. Thus in the BB framework the objective function that generations (or a social planner for the whole dynasty of various generations) is imagined to solve continues to be a discounted sum of period utilities which then carries the interpretation of *dynastic* rather than individual lifetime utility function. Let us set up the model.

The economy consists of identical individual households. Each individual in a household lives for two generations (childhood and adulthood), indexed by  $t$ , and thereafter ceases to exist. Individuals can, however, beget children at the beginning of their adult stage; these children are exactly like their parents in preferences over consumption, and also live two generations. (These assumptions clearly abstract from considerations of birth and death probabilities.) Individuals are assumed to be altruistic with respect to the utility of their direct progeny. Formally, Barro and Becker assume that

the utility  $U_t$  of an adult of the  $t$ th generation is given by

$$(6.1) \quad U_t = v(c_t) + \alpha(n_t)n_t U_{t+1}$$

where  $v(c_t)$  is the period utility component due to current (own) consumption of the single commodity,  $n_t$  is the number of children generated by this individual, and  $\alpha(n_t)$  is a function which captures the degree of altruism attached to each of the  $n_t$  progeny. In their model, Barro and Becker specialize  $\alpha$  to be:

$$\alpha(n_t) = \alpha n_t^{-\epsilon} \quad ; \quad 0 < \alpha < 1, 0 < \epsilon < 1.$$

The product  $\alpha(n_t)n_t = \alpha n_t^{1-\epsilon}$ ; given the range of  $\epsilon$ , this implies that  $U_t$  is increasing in the number of children  $n_t$  for fixed  $U_{t+1}$ , but the marginal increments to  $U_t$  are decreasing. Recursively substituting  $U_{t+j}$  in equation (6.1) beginning from  $t+j = 1$  yields the *dynastic utility function*

$$(6.1') \quad U_0 = \sum_{t=0}^{\infty} \alpha^t N_t^{1-\epsilon} v(c_t), \text{ where } N_t = \prod_{j=0}^{t-1} n_j.$$

The variable  $N_t$  is just the number of descendants in generation  $t$ . To make for convenient closed-form solutions, Barro and Becker specialize the function  $v$  to be

$$v(c_t) = c_t^{\sigma} \quad ; \quad \sigma < 1.$$

The second component of the BB model is the wealth constraint of an adult in generation  $t$ . It is assumed that labor supply to the market is fixed and normalized to 1 unit and that the wage rate the adult individual faces is  $w_t$ . The individual inherits from her immediate ancestor a bequest of  $k_t$  units of (human or nonhuman) capital and can leave similar bequests to each child of (human or nonhuman) capital  $k_{t+1}$  units. Capital  $k_t$  earns a rental rate,  $r_t$ . Next, let  $\beta_t$  be the per-child cost of raising children of generation  $t$  into adulthood. The wealth constraint facing the two-period lived individual is then:

$$(6.2) \quad w_t + (1+r_t)k_t = c_t + n_t(\beta_t + k_{t+1})$$

The consumption sub-problem of the BB model is taken from the point of view of

the dynastic head at time 0. She is assumed to choose for all  $t \geq 0$  consumption levels  $c_t$ , capital per child  $k_{t+1}$ , and the number of progeny of each adult in generation  $t$ ,  $n_t$ , to maximize (6.1') subject to the sequence of wealth constraints (6.2) that each subsequent generation will face and her initial inheritance  $k_0$ . It is assumed that the dynastic head knows for sure the preferences and prices that each subsequent adult of generation  $t$  will face, so that her optimal choices as dynastic head at  $t=0$  will continue to be optimal for decision-makers in later generations  $t=1,2,\dots$  (i.e., the dynastic head's optimal plans are "time-consistent.") After some algebra, the maximization of (6.1) subject to (6.2) and  $k_t > 0$  yields the first-order conditions

$$(6.3a) \quad \left[ \frac{c_{t+1}}{c_t} \right]^{(1-\sigma)} = \alpha n_t^{-\epsilon} (1+r_{t+1})$$

$$(6.3b) \quad c_t = \left[ \frac{\sigma}{1-\epsilon-\sigma} \right] [\beta_{t-1}(1+r_t) - w_t]$$

$$(6.3c) \quad k_0 + \sum_{t=0}^{\infty} [\Pi_{j=0}^t (1+r_j)^{-1}] N_t w_t = \sum_{t=0}^{\infty} [\Pi_{j=0}^t (1+r_j)^{-1}] (N_t c_t + N_{t+1} \beta_t)$$

Equation (6.3a) and (6.3b) give the optimal choice for consumption growth and fertility. Looking at (6.3b) in particular shows the effect of higher child-raising costs on children's consumption. As the cost of raising a child in period  $t-1$   $\beta_{t-1}$  goes up (BB assume that  $1-\epsilon-\sigma > 0$ ), making up for this higher cost requires raising the parent's own utility benefit from children. For fixed  $n_t$ , this is done by raising children's consumption levels. Also from (6.3b) we see that consumption per person can only grow between  $t$  and  $t+1$  if the cost of raising children expressed in period  $t$  consumption goods, i.e.,  $\beta_{t-1}(1+r_t)$ , net of their future income  $w_t$ , is growing.

To see now what determines optimal fertility, substitute for  $c_t$  in (6.3a) using (6.3b) to get

$$(6.4) \quad n_t^{\epsilon} = [\alpha(1+r_{t+1})] \cdot \left[ \frac{\beta_{t-1}(1+r_t) - w_t}{\beta_t(1+r_{t+1}) - w_{t+1}} \right]^{(1-\sigma)}$$

Fertility is thus an increasing function of altruism  $\alpha$  and the real interest rate  $r_{t+1}$ . This is because higher altruism or real interest rates make it more economical to shift away consumption into the future by having more children. As BB remark:

$$(6.13') \quad \bar{c} = \left[ \frac{\sigma}{1-\epsilon-\sigma} \right] \left[ \beta \cdot \frac{(1+r)}{(1+g)} - \bar{w} \right]$$

$$(6.14') \quad \bar{w} + (1+r)\bar{k} = \left[ \frac{\sigma}{1-\epsilon-\sigma} \right] \left[ \beta \cdot \frac{(1+r)}{(1+g)} - \bar{w} \right] + n[\beta + (1+g)\bar{k}] + bn\bar{k}[1+r-(1+g)n]$$

$$(6.15') \quad n^\epsilon = [\alpha(1+r)] \left[ \frac{1}{(1+g)} \right]^{(1-\sigma)}$$

where all variables are now taken at their steady-state values. It is this last set of equations upon which the interplay between fertility and macroeconomic variables is based. We examine this in the next section.

### 6.3. Discussion.

Before discussing the theoretical results, let us isolate some distinctive features of the BB model.

- i. Individuals are altruistic about their children's consumption. This altruism is reflected in the rate at which future consumptions are discounted.
- ii. The BB model assumes perfect fertility control so that realized fertility is unilaterally determined by the generational demand for children. The model abstracts from contraceptive practices, biological factors, and other supply-side determinants of family size in a generation.
- iii. All births are modelled to take place at the same time (the beginning of the adult stage of life.) Hence the model abstracts from issues of birth timing or spacing.
- iv. In relation to (ii) there is no uncertainty at all in the model. Individuals live for exactly two periods. There is no uncertainty about the occurrence of a birth or a death. There are no shocks to preferences, or unobservable heterogeneity in individual fecundity.
- v. Wealth and capital accumulation are integral parts of the model, however,

intertemporal wealth are essentially bequests to children. Hence the utility return to adding to children's wealth is what is equated at the margin to the value of parent's sacrifice of current consumption.

- vi. The model features a competitive production sector characterized by constant returns to scale and exogenous labor-augmenting technological progress of the Harrod-neutral type. This assumption allows for steady-state growth of the capital stock per person  $k_t$  at the exogenous rate  $g$ .
- vii. There is no labor-leisure tradeoff in this model. Leisure time is assumed fixed. The tradeoff is between the total amount of time devoted to child-raising and the total amount of time devoted to market labor. The per-child time requirements for raising children are fixed at  $b$ , however growth in wage rates (the opportunity cost of nonmarket time) due to productivity growth leads to growth in total child-rearing costs.
- viii. The BB model is a closed-economy model with no migration of labor or capital, and no trade or specialization in production.

The consequences of the BB assumptions are many, and the authors go into a detailed discussion of the implications of the steady-state equations (6.12')–(6.15') for macroeconomic variables. Repeating and reviewing all of these would be unduly tedious so we restrict attention to those implications having to do with fertility and fertility-related variables such as child costs. Further, in the discussion that follows relationships between variables are to be understood as steady-state relationships. Therefore let us focus initially on the optimal steady-state fertility equation (6.15'). In the discussion that follows.

- i. The number of children per adult individual is constant. Population, however, grows exponentially once the steady-state is reached; beginning with a single dynastic head at  $t=0$ , if the economy attains the steady-state immediately the first generation will have  $n$  individuals, the second  $n^2$ , etc.
- ii. Fertility is increasing with altruism and the real interest rate and decreasing in the rate of growth of per capita consumption  $(1+g)^{(1-\sigma)}$ . That fertility should increase with altruism suggests that more altruistic individuals or cultures will



The change in the relative number of people across generations is a form of intertemporal substitution that replaces the usual response in the path of consumption per person. (Barro and Becker, 1989, p.485.)

Moreover, the inverse of the term in parenthesis in (6.4) is the growth of the net costs of children. Thus fertility in the BB model is also a (negative) function of the growth of net costs of children.

Equation (6.3c) on the other hand is a dynastic budget constraint which essentially says that the present value of income flows equals the present value of all expenditures. While this equation sheds some light on the intergenerational discount rate we do not discuss this here.

To complete the model of a macroeconomy BB now superimpose a production sector on (6.1)–(6.2). Assume now that there is a single firm whose role is to produce the consumption commodity in each period according to an aggregate production function of the form:

$$(6.5) \quad Y_t = F(K_t, (1+g)^t L_t)$$

where  $Y_t$  is aggregate output;  $F$  is a constant returns to scale function, i.e.,  $\lambda F_t = F(\lambda K_t, \lambda (1+g)^t L_t)$  for any  $\lambda > 0$ ;  $K_t$  and  $L_t$  are quantities of capital and labor employed by the firm in production, and the term  $(1+g)^t$  is a measure of exogenous Harrod–neutral technical progress (one can think of this as measuring exogenous growth in the quality or productivity of labor.)

The choice of the Harrod–neutral form for technical growth was made in order for the model to have steady–state growth, in which all variables will be growing at the rate  $g$ . The constant returns to scale assumption allows one to divide both sides of the production function by  $(1+g)^t L_t$  to get outputs and capital inputs measured in units per "effective" worker:

$$(6.6) \quad \bar{y}_t \equiv \frac{Y_t}{(1+g)^t L_t} = \frac{F(K_t, (1+g)^t L_t)}{(1+g)^t L_t} = F\left[\frac{K_t}{(1+g)^t L_t}, 1\right] \equiv F(\bar{K}_t, 1)$$

so that, writing  $f(\bar{K}_t) = F(\bar{K}_t, 1)$ , the production function for "effective" output is

$$(6.5') \quad \bar{y}_t \equiv f(\bar{K}_t)$$

$\bar{y}_t$  and  $\bar{K}_t$  now carry the interpretation of output per effective worker and capital per effective worker. Suppose that the firm takes as given the wage rate  $w_t$  and the rental rate of capital  $r_t$ . Standard profit-maximizing conditions are known to be

$$(6.7) \quad r_t = f'(\bar{K}_t) ; \quad w_t = [f(\bar{K}_t) - f'(\bar{K}_t)\bar{K}_t](1+g)^t$$

These conditions, together with the wealth constraint (6.2), will be used later to determine the evolution of capital  $k_t$ . For definitional purposes it will later be useful to work with the wage rate per effective worker defined as

$$(6.8) \quad \bar{w}_t \equiv \frac{w_t}{(1+g)^t L_t} = [f(\bar{K}_t) - f'(\bar{K}_t)\bar{K}_t]$$

To close the BB model requires us to describe how labor  $L_t$  gets to be supplied to the firm. Assume, as did Barro and Becker, that some fixed amount  $b$  of the time endowment of adult individuals must be used towards child rearing in addition to other non-time (commodity) inputs. If  $w_t$  is the wage rate the total time cost of raising a child will be  $bw_t$ . In this economy the wage rate  $w_t$  (not  $\bar{w}_t$ ) will be growing over time at the rate  $g$  by equation (6.7). For non-time (commodity) costs of child-raising not to become insignificant as the economy grows, the commodity costs of child-rearing should also grow at rate  $g$ . So let  $a$  be the fixed quantity of the single commodity applied towards the raising of a child. The total cost of raising a child which was labelled  $\beta_t$ , may then be written as

$$(6.9) \quad \beta_t = a(1+g)^t + bw_t$$

and, again, for later convenience we may define the "effective cost" of raising a child to be

$$(6.10) \quad \beta_t \equiv \frac{\beta_t}{(1+g)^t L_t} = a + b[f(\bar{K}_t) - f'(\bar{K}_t)\bar{K}_t].$$

Since the time endowment of the adult individual is normalized to 1, the labor supply available for production equals  $1 - bn_t$  times the number of people alive at  $t$ :

$$(6.11) \quad L_t = (1 - bn_t)N_t$$

We will be rewriting, very shortly, variables appearing in the wealth constraint and the optimality conditions in terms of effective labor units. This last equation implies that the variable  $k_t$  which is measured in terms of capital per (adult) person will need to be divided by  $(1 - bn_t)(1 + g)^t$  to get its counterpart  $\bar{K}_t$  measured in terms of effective (adult) laborers.

## 6.2. Solution.<sup>13</sup>

Let us now provide the solution of the model. First express the variables of remaining optimality conditions in terms of effective workers. The wealth constraint may be rewritten using  $(1 - bn_t)\bar{K}_t = (1 + g)^{-t}k_t$  and  $\bar{c}_t = (1 + g)^{-t}c_t$ :

$$(6.12) \quad \bar{w}_t + (1 - bn_t)(1 + r_t)\bar{K}_t = \bar{c}_t + n_t[\beta_t + (1 + g)(1 - bn_{t+1})\bar{K}_{t+1}]$$

Rewriting (6.3b) in similar fashion gives

$$(6.13) \quad \bar{c}_t = \left[ \frac{\sigma}{1 - \epsilon - \sigma} \right] \left[ \beta_{t-1} \cdot \frac{(1 + r_t)}{(1 + g)} - \bar{w}_t \right]$$

These last two equations in turn imply an expression that governs the evolution of capital per effective worker  $\bar{K}_t$ . As it turns out, fertility  $n_t$  matters for capital accumulation:

$$(6.14) \quad \bar{w}_t + (1 - bn_t)(1 + r_t)\bar{K}_t = \left[ \frac{\sigma}{1 - \epsilon - \sigma} \right] \left[ \beta_{t-1} \frac{(1 + r_t)}{(1 + g)} - \bar{w}_t \right] + n_t[\beta_t + (1 + g)(1 - bn_{t+1})\bar{K}_{t+1}]$$

To see why this equation determines the evolution of  $\bar{k}_t$ , note that it implicitly gives  $\bar{k}_{t+1}$  as a function of  $\bar{k}_t$  and variables dated  $t$  and earlier, except for the variable  $\bar{n}_{t+1}$ . However, from (6.4) we find that  $n_t$  is a function of the price variables  $\bar{w}_t, r_t$ , and  $\beta_t$ :

$$(6.15) \quad n_t^\epsilon = [\alpha(1+r_{t+1})] \left[ \frac{1}{(1+g)} \cdot \frac{\beta_{t-1}(1+r_t) - (1+g)\bar{w}_t}{\beta_t(1+r_{t+1}) - (1+g)\bar{w}_{t+1}} \right]^{(1-\sigma)}$$

Since  $\bar{w}_t, r_t$ , and  $\beta_t$  are in turn functions of  $\bar{k}_t$  only (see equations (6.7), (6.8), and (6.10) above) updating (6.15) and substitution into equation (6.14) makes (6.14) a function of  $\bar{k}_t, \bar{k}_{t+1}$ , and parameters only. The procedure for studying capital accumulation is straightforward: given that  $n_0=1$ , and a starting value for  $k_0$ , we use this starting value in (6.7), (6.8), and (6.10) to find  $r_1, w_1$ , and  $\beta_1$ . From this we can calculate by (6.15) the optimal number of descendants  $n_1$ . We then obtain  $k_1$  from (6.14). Iterating this procedure for  $t > 1$  gives  $k_{t+1}$  for  $t > 1$ .

Rather than study all the possible paths along which  $\bar{k}_t, \bar{n}_t, \bar{c}_t$ , and  $\bar{y}_t$  evolve, Barro and Becker focus only on *the steady-state growth path*. This is the path along which capital per effective worker is constant, that is,  $\bar{k}_{t+1} = \bar{k}_t$ . This path is usually interpreted as the path that the economy will converge to in the long-run. As Barro and Becker prove, this steady-state exists and is unique if the parameter  $b$  is not "too large." For details on these results, see the original source.

Since  $\bar{k}_t$  is constant along this path, so will  $r_t, \bar{w}_t, \beta_t$  be constant. From this and equations (6.3b), (6.5'), and (6.15) above it follows that  $\bar{y}_t, \bar{c}_t$ , and also  $n_t$  will not grow over time. However if  $\bar{k}_t$  is steady, this does not mean that  $k_t$  is also steady, since  $k_t$  is capital per person, not per effective worker. While  $\bar{k}_t$  is fixed,  $k_t$  will be growing each period by the factor  $(1+g)$ . By the same token, while  $\bar{w}_t, \bar{c}_t$ , and  $\bar{y}_t$  are steady, their counterparts measured in per-person terms,  $w_t, c_t$ , and  $y_t$ , are growing at the rate  $g$  in the steady-state. Taking this into account, assuming that the BB economy has reached a long-run steady-state growth path, (6.12)–(6.15) simplify to

$$(6.12') \quad \bar{w} + (1+r)\bar{k} = \bar{c} + n[\beta + (1+g)\bar{k}] + bn\bar{k}[1+r-(1+g)n]$$

eventually have larger steady-state populations, reflecting a kind of "natural selection" process. That fertility should increase with the real interest rate makes sense in the model, since increasing the relative value of consumption in later periods is essentially increasing the value of children alive in later periods. That it should decrease as consumption per capita grows faster is a consequence of the discount rate for future consumption depending positively on the number of children in those future periods. Observe that in the steady-state the utility discount rate between two adjacent periods is  $\alpha n^{1-\epsilon}$ . Unlike standard macroeconomic growth models, the discount rate in BB depends on the *endogenously* determined fertility level. In order to encourage higher levels of future consumption (hence, higher consumption growth) one has to lower the discount rate of future consumption. This can be done by having more children.

Also worth considering is the effect of an increase in effective child-rearing costs  $\bar{\beta}$  on fertility. Normally it is expected that higher  $\bar{\beta}$  would tend to lower fertility levels. In the BB model, however,

- iii. Increases in child-rearing costs *have no direct effect* on steady-state fertility levels  $n$ . (Child-rearing costs  $\bar{\beta}$  do not appear at all in equation (6.15').) Any effect of  $\bar{\beta}$  on  $n$  is indirect, working through changes in the steady-state levels of  $\bar{K}_t$  and  $r$ . We have argued above that in the BB model higher  $\bar{\beta}$  only motivates individuals to endow children with more consumption  $c_t$ , and this is also true for the steady state  $\bar{c}$ . To raise steady-state  $\bar{c}$ , however, requires one to either increase  $\bar{y}$ , which is possible only by raising  $\bar{K}$ . However raising  $\bar{K}$  reduces steady-state real interest rates  $r$  because, by (6.7),  $r = f'(\bar{K})$ . The reduction in  $r$  makes  $n$  decline, by result (ii) above.

The empirical content of (iii) is as follows: if the model is true, then policy or exogenous shocks that bring up the costs of raising children should have the effect of raising both real interest rates and fertility. *However, if one were to eliminate that component of fertility which is explained by changes in real interest rates, the residual should be orthogonal to the policy or exogenous shock.* Examples of policies that would have this

effect are direct taxes on children, higher social security taxes, eliminations of child-maintenance related subsidies and exemptions, legislated increases in wage rates. Examples of other shocks would be structural shifts in female labor force participation rates, experience-related changes in wage rates, or even heterogeneity across families within the economy or across economies.

Other results of interest are:

- iv. Whereas in (ii) it was remarked that higher  $g$  tends to have the direct effect of reducing fertility  $n$ , There may be an indirect effect of  $g$  that works in the opposite direction, operating through increases in  $r$ . As long as the steady-state is stable, BB show that higher per capita growth  $g$  tends to reduce  $\bar{K}$  and hence raise  $r$ . Therefore the overall effect of  $g$  on fertility  $n$  is ambiguous.
- v. Higher taxes on capital may or may not result in higher  $n$ . This is because higher taxes on capital tend to reduce  $\bar{K}$ , however individuals may reduce  $\bar{K}$  by either increasing  $n$ , or decreasing  $n$  but decreasing  $K_t$  at a faster rate than the resulting reduction in  $L_t$ .

Finally, we should mention for completeness that fertility decisions do provide an endogenous source of growth of *total output* in the model. In the steady-state, output per effective worker is steady, but output per "ordinary" worker grows at the rate  $g$ . However the pool of ordinary workers grows at the same rate as the population, which is increasing by the factor  $n$  with each generation. Therefore total output itself should be growing by a factor of  $(1+g)n$  each period.

#### 6.4. Econometric Implementation.

The BB model provides a host of testable relationships between fertility, capital accumulation, child-rearing costs, real interest rates, and exogenous sources of growth. To our knowledge both the model itself and its implications have yet to be econometrically examined. At this point we wish only to make three general comments on developing an estimation strategy for the model.

First, the original BB model is nonstochastic; in order then to estimate the model structurally the analyst has to make assumptions about the type and form of the stochastic

influences that are believed to impinge upon the model. Naturally, the way in which stochastic shocks are specified matters for the identification and robustness of the model parameters, as well as the appropriate econometric technique to employ.

Next, we remark that while the BB model may be applied to both time-series data and cross-sectional data, there are issues of aggregation and the size of the resulting dataset. With time-series data, the main question is how long a time period is necessary for generational effects to be manifest. Barro and Becker (1988) seem to indicate that a period is typically long, probably around thirty to forty years. Once the period length is determined one has to then aggregate up the data. This time-series approach thus requires a very extensive set of historical data to implement, as the sampling frequency of the data may be as much as thirty or so years. For purely cross-sectional datasets the BB model may not be appropriate for purely cross-sectional data sets if the level of aggregation of the individual data is at an open-economy level, rather than a closed-economy one. (For example, analyzing U.S. data at the state level may require modification of the BB model to account for effects of interstate capital flows and labor migration on capital accumulation and child production.) Cross-sectional data over several generations is probably the best one can realistically hope for, but even this is not entirely free of limitations. As mentioned in Barro and Becker (1989) aggregation across individuals may smooth out some of the steady-state convergence patterns at the household level, making it difficult, if not impossible to analyze these generational patterns.

Lastly, we should mention that not all empirical work in macroeconomic growth models has relied on structuralist econometric approaches. The analyst may prefer a structural econometric methods such as vector autoregressions, cointegration methods, and related time-series frameworks to understand the generational patterns of fertility as long as the data permits. Alternatively, the analyst might employ simulation or calibration methods to determine how well a parameterized version of the model fits the data. See Smith and Gregory (1990) for a discussion of these methods as they have been used in macroeconomics. Like traditional econometric approaches, these strategies would require explicit specification of the stochastic elements of the model.

## 6.5. Evaluation.

The Barro and Becker model integrates fertility choices into a macroeconomic model of growth. This synthesis is a particularly useful framework for the analysis of aggregate movements in fertility across closed economies over long spans of time. It is

especially relevant for addressing questions about how fertility affects long-run growth and capital accumulation. As a model of the determinants of fertility it emphasizes economic factors acting mainly through the demand for fertility, specifically, things like the real interest rate, the degree of altruism, and the costs of child-rearing. It is therefore much less useful for the analysis of other determinants of fertility that act upon the supply of births, in particular, contraceptive use, individual fecundity, health inputs, and biology. On the face of it, it would seem that some of these other determinants can be introduced into the BB model without upsetting their basic results on capital accumulation and the other variables tied to capital accumulation. The level at which fertility choices are modelled abstracts from life-cycle considerations and examines intergenerational choice. In its present form it is not yet useful, then, for the analysis of birth spacing and timing patterns within a generation. Barro and Becker indicate that the model can be extended to incorporate life-cycle decisions, but not without creating substantial complications.

While the BB model is mainly useful for understanding the implications of fertility choice on the time paths of key macroeconomic variables, it is also of direct interest to empirical research on fertility because it does provide unique testable propositions concerning the effects of policy on fertility. On this point, it is of some importance to note that over the long-run the BB model predicts that influences of policy on generational fertility act only *indirectly* through three channels: altruism, consumption growth, and real interest rates. This is in stark contrast to most other models of fertility which typically derive *direct structural connections* between policy or costs of children and the demand for children.

We do not view the BB model as a direct alternative to the micromodels of Wolpin or Newman. Rather, we view the BB model as more of an integrating general equilibrium framework, which can be modified to accommodate features of micromodels, if the nature of the research questions so requires. While much existing research into fertility has emphasized the analysis of microeconomic life-cycle decisions, the time may be right to integrate this knowledge of the fertility decision with its consequences for growth and fluctuations in the modern macroeconomy.

### ***Conclusions.***

The objective of this paper was to survey critical approaches to the economic modelling of fertility determinants. We have identified two distinct frameworks: static choice models of lifetime fertility levels and dynamic models of birth timing, birth spacing, and intergenerational fertility levels. Within each framework we have reviewed three



distinct prototypical methodologies for the analysis of the fertility question. Our belief is that these six models and their estimation technologies will continue to remain influential for some time to come.

This paper has not examined in great specificity the many ways in which policy inferences becomes limited or biased or even erroneous due to the use of one methodological framework over another. To provide a rigorous treatment of these issues was beyond the intended scope of this paper, and requires a separate study. Moreover, we did not attempt to survey in detail or critique *empirical findings* of studies that have used each model. However we have tried to incorporate, at relevant points, discussions of how the models surveyed limit the scope of the policy questions that one can ask, and how one might examine particular policy questions in the context of these models.

### *Notes.*

<sup>1</sup>See Easterlin's rebuttal to Schultz in Easterlin, 1986, p.3.

<sup>2</sup>An example is the Livingston panel survey, which polls the expectations of economists about the future levels of prices in the economy, and other variables.

<sup>3</sup>Two concepts in economics which have similar problems of interpretation are the concepts of "voluntary" and "involuntary" unemployment.

<sup>4</sup>In the reformulation of Montgomery (1987) some of this is lost so that the Montgomery model discussed below cannot be properly regarded as a path model.

<sup>5</sup>An excellent and detailed reference on the Tobit estimation procedure, together with the related probit and logit procedures is Maddala (1983).

<sup>6</sup>Incidentally, Easterlin's interpretation of Malinvaud in the opening quotation to his 1986 rejoinder is not quite to the point either. Malinvaud's quotation ( "The methods of mathematical statistics do not provide us with a means of specifying the model.") says that fishing in your pool of data will not provide a specification for the model, only clues as to what specifications are likely to be rejected by that particular data set. It is the job of theory to provide specifications that can be tested against a broad set of real world data. The theory whose implications secure the widest support becomes the model that is tentatively accepted as "truth," that is the position taken in this review. But to say, as Easterlin suggests, that "the choice of model has nothing to do with the statistical methodology," (Easterlin, 1986, pp.13-14.) could mean that either (1) a theoretical model cannot lead one to some particular empirical specifications, requiring in turn their own appropriate statistical procedures, or (2) that theorizing is not conditioned by empiricism in one form or another. Neither of these is true generally, and neither of these statements appears to be what Malinvaud meant.

<sup>7</sup>Dependence of utility functions on outside forces that are not influenced by choice behavior is, to a degree, manageable. In a crude way this is what one does in resorting to the use of dummy variables in a regression to capture the dependence of preferences on outside social or psychological factors. However, problems involving the dependence of utility functions on choice behavior itself are much more difficult, as the classic problem of moral hazard illustrates.

<sup>8</sup>To be fair economists are really quite silent on whether all behavior is in fact a conscious process or something else—There is more general agreement, however, that whether or

not individuals behave consciously, observed economic behavior is such that one can proceed to model individuals behaving *as if* they made conscious choices. The act of driving a car at some point becomes instinctive behavior, but one can *model* the driving behavior of an individual as a solution to a set of equations describing the velocity and acceleration of the car, the speed of the person's reflexes, the number of cars on the same road, his utility or disutility of pain, etc., without really believing that individuals actually solve such an equation system in their heads before making a decision to apply the brakes or not to.

As applied to the fertility decision, the economic approach to modelling suggests that whatever be the process or the variables that people use to decide on whether to have an additional child or not—instinct, racial memory, medicine men, calculus of variations—observed behavior is such that we may as well have chosen to describe the choice process *as if* people consciously maximized a utility function.

If anything, from casual observation alone we might form strong priors that fertility choices are likely to be remarkably conscious and deliberate in both developed societies where the economic opportunity costs of fertility is high and in less developed societies where income flows are low enough or highly uncertain that children are likely to be regarded as investments or a form of social security. The contraceptive method choice is also likely to be consciously undertaken with explicit and psychic costs, perceived method-specific failure rates, and long-term health effects figuring significantly in the selection process. The problem in less developed country scenarios is not whether individuals consciously or unconsciously form their decisions about how many children to have or what methods to use. The problem is whether the fact that information about things like natural fecundity or the reliability of contraceptive methods is poor and costly to gather, and the fact that there is simply more randomness about the future environment make it useless to *model* individuals as maximizing agents. The best way to answer this question is to fit an economic model and see if it improves upon the results of models without maximizing individuals.

<sup>9</sup>RS anticipate this in a footnote (no. 10) and they had actually conducted Hausman specification tests against OLS estimates of (2.3'), but do not report similar test results for the two-stage instrumental-variables estimates. Presumably(?) the two-stage model passed this specification test for in the sequel RS claim that its estimates are consistent. In a later study using Malaysian data (Rosenzweig and Schultz, 1987) they do report results of Hausman tests for the second-stage estimates.

<sup>10</sup>Montgomery recognizes this: "How does the framework described here differ from that

of Easterlin and Crimmins? Note that the coefficient  $\alpha_2$ , reflecting the psychic costs of contraception, is an element of the demand-for-births equation (6) [i.e., equation (3.6a) in the sequel above.] The greater the disutility associated with contraception, the higher the derived demand for births. Easterlin's hypothetical concept of demand—the demand for births under the condition that contraception is costless—could certainly be represented by equation (6) with  $\alpha_2$  set to zero. That theoretical representation, however, achieves little in the absence of an empirical *measure* of notional or hypothetical demands." (Montgomery, 1987, p.483.)

<sup>11</sup>Or basic determinants, if one likes—in Montgomery's paper these are marriage duration, education, region ethnicity, and survival probabilities calculated from independent census data.

<sup>12</sup>In the more recent model of Hotz and Miller, a discrete-time dynamic programming framework is developed which considers the impact on childbearing of the potential to participate in the labor force. Contraception influences the conception probability, but is essentially a zero-one choice for the individual. Unlike the Wolpin framework which generates spacing of births due to the interplay of discounting and the rising time profile of income, Hotz and Miller's framework generates spacing due to income changes, as well as reductions in the time cost of children as children age.

<sup>13</sup>This section may be skipped for those interested mainly in the analytical results. The main conclusion of this section is that there is a unique steady-state for the BB model in which  $\bar{k}_t, \bar{w}_t, \bar{\beta}_t, \bar{r}_t, \bar{y}_t, \bar{c}_t$ , and  $n_t$  are constant (but  $k_t, w_t, \beta_t, y_t$ , and  $c_t$  are growing at the rate  $g$ .) The steady-state relationships between these variables are given by equations (6.12')–(6.15').

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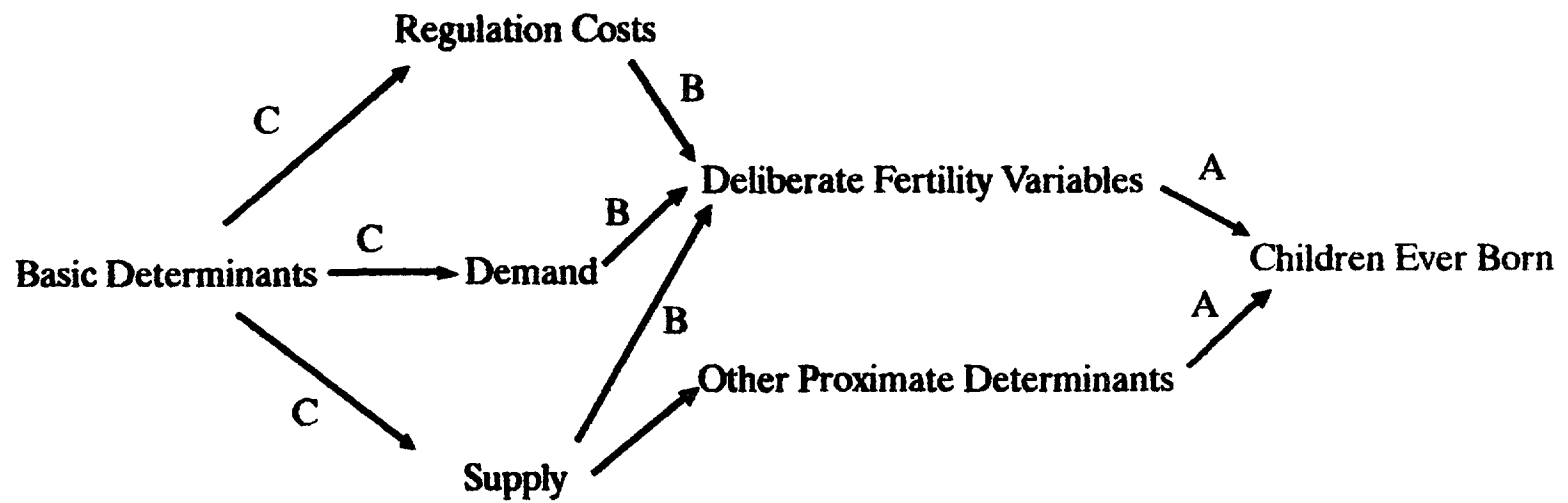


Figure 1.



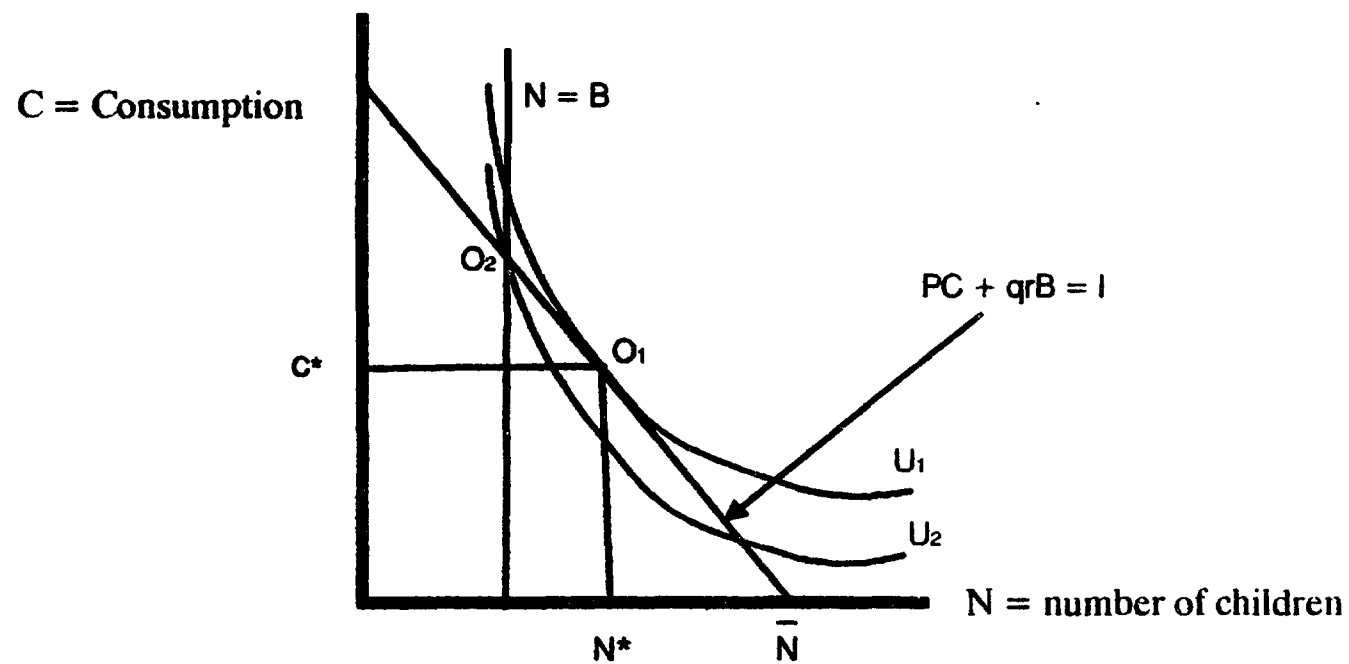


Figure 2.

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